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MEASUREMENTS ON THE INDEX OF REFRACTION OF AIR FOR WAVE LENGTHS FROM 2218 A TO 9000 A

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I. INTRODUCTION

The optical property of air, by virtue of which the velocity, wave length, and, in general, the direction of light is changed when passing from other media into air, has been of practical interest to astronomers and physicists for many centuries. This interest has resulted in a large number of experimental and theoretical researches which have reacted to create a still greater interest in the subject. The majority of researches on the refractivity of air received their inspiration from the desire to correct astronomical observations for the refraction of the earth's atmosphere, and in this connection much work was done to establish the dependence of such corrections on the state of the air as determined by its temperature, pressure, and constitution. Other investigations were later undertaken more especially to test certain theories and laws proposed to represent the relation of refractivity to density and the connection between refractivity and wave length. These are of primary importance to spectroscopists and other physicists. A brief review of the objects, experimental methods, and results of these earlier investigations will assist in explaining the purpose of this additional investigation on the index of refraction and dispersion of air described in detail in this paper.

1. ASTRONOMICAL OBSERVATIONS

The refractive power of air was first recognized 1900 years ago by Cleomedes, who suspected that it explained the phenomenon of a lunar eclipse when both sun and moon were visible above the earth's horizon. Ptolemy, in the second century, expressed correct ideas of astronomical refraction in his Optics. His attempts to measure this refraction and to find the law connecting incidence and refraction angles were unsuccessful. Astronomical refraction was of no practical importance until the observational instruments and methods were sufficiently refined to measure minutes of arc. In fact, Tycho Brahe,[1] in the sixteenth century, was the first to measure this refraction successfully and he constructed the first tables of corrections to astronomical observations. After the discovery in the seventeenth century of Snell's law that the index of refraction is a constant ratio of the sines of the incidence and refraction angles, theoretical researches attempted to find the differential equation of the path of light in the earth's atmosphere.

[1] Tyco Brahe, Astronomie instauratae Progymnasmata, Francofurt, 1648.

Such an equation appears to have been first derived by Brook Taylor [2] in 1715. Then followed the work of Cassini, Bouguer, Bernoulli, Simpson, Bradley, Euler, Lambert, Lagrange, Kramp, and others, climaxed by that of Laplace in 1805. Tables constructed on Laplace's formulae were used in astronomy and geodesy for many years. A complete review of these labors of astronomers and physicists upon this complex but interesting subject of astronomical refraction is given in a prize essay by C. Bruhns.[3]

The earliest absolute value of the refractivity of air was determined by Delambre,[4] whose result was obtained in 1805 from more than 500 observations of the altitudes of circum polar stars. Later values have been deduced from astronomical observations by Bessel,[5] Gylden,[6] Fuss,[7] and others.

2. PRISM METHOD

The prism method for measuring the refractive power of gases was first used by Biot and Arago[8] (1806), who observed with a telescope the apparent change of position of an object seen through a hollow prism filled with air when the density of the air was changed by varying the temperature from -1.5 to 25° C and the pressure from 0.005 to 0.8 m of mercury. From these experiments the absolute index

$$n = \frac{\sin i}{\sin r}$$

was obtained for white light and it was concluded that the refractivity of air was proportional to the density. Dulong[9] (1826) also measured the index of refraction of air for white light by the prism method. Later (1893) Kayser and Runge[10] used a copper prism with quartz windows and placed the prism so as to cover part of a concave grating. Lines in the iron spectrum and in the zinc spectrum were photographed, some of the light passing by the prism and the rest passing through the prism containing air under pressures from 1 to 10 atmospheres. The refractive

[2] Brook Taylor, Methodus incrementum, London, 1715.
[3] C. Bruhns, Die Astronomische Strahlenbrechung, Leipzig; 1861.
[4] Delambre, s. Laplace Mecanique celeste, 4; 1805.
[5] Bessel, s. Biot Compt. rend., 40.
[6] Gylden, Mem. de l'acad. de St. Petersburg, 10, No. 1, 1866, and 12, No. 4, 1868.
[7] Fuss, ibid., 18, No. 4; 1868.
[8] Biot and Arago, Memoires de l'Institute, 7, p. 301; 1806.
[9] Dulong, Ann. de Chim. et de Phys. (2), 31, p. 154; 1826.
[10] Kayser and Runge, Ann. d. Phys. 50, p. 293; 1893.

indices for different wave lengths were then determined from the measured shifts of the spectrum lines due to light passing through the prism. Wet air at room temperatures from 11 to 19° C was used for these experiments and the results reduced to represent air at 0° C and 760 mm.

A hollow glass prism with air under pressures from 1 to 5 atmospheres was used by Carnazzi.[11] Indices of refraction were determined from measurements of the prism angle and the incidence and emergence angles. These results indicated an increase of about 15 per cent in $\frac{n-1}{d}$ for this pressure range.

Lang [12] investigated the dependence of refractive index of air on temperature by observing beams of light reflected from prism faces through windows of a heating apparatus surrounding the prism and noting the deviations for various temperature differences (2.4 to 95.0° C) between the air inside and outside the apparatus. The observations were represented by the equation $n_t = n_0 - 0.000000905t + 0.00000000235t^2$.

3. INTERFEROMETER METHODS

The interferometer method for the measurement of refractive indices of gases was first applied by Jamin [13] (1857). Two Fresnel mirrors were used to make two beams of light, traversing tubes 1 m long, interfere and the difference in optical path produced by changing the pressure in one of the tubes was determined with a compensator. Modifications of Jamin's interferometer served later for the work of Ketteler,[14] Lorenz,[15] Mascart,[16] Chappuis and Riviere,[17] Perreau,[18] Walker,[19] Gale,[20] Magri,[21] C. and M. Cuthbertson,[22] Kaiser,[22] Gruschke,[24] Ahrberg,[25] and Koch.[26] Ketteler measured the absolute index of ordinary air at 0° C and 760 mm for sodium light and indices for lithium and thallium light

[11] Carnazzi, Il Nuovo Cimento (6), p. 385; 1897.
[12] Von Lang, Pogg. Ann., 153, p. 448; 1874.
[13] Jamin, Ann. de Chim. et de Phys., 49, p. 282; 1857.
[14] Ketteler, Pogg. Ann., 124, p. 390; 1865.
[15] Lorenz, Wied Ann., 11, p. 70; 1880.
[16] Mascart, Compt. rend., 78, p. 617; 1874.
[17] Chappuis et Riviere, Ann. de Chim. et de Phys. (6), 14, p. 5; 1888.
[18] Perreau, Ann. de Chim. et de Phys. 7, p. 289; 1896.
[19] Walker, Phil. Trans., 201, p. 435; 1903.
[20] Gale, Phys. Rev., 14, p. 1; 1902.
[21] Magri, Phys. Zeitschr., 6, p. 629; 1905.
[22] C. and M. Cuthbertson, Proc. Roy. Soc., 83, p. 151; 1910.
[23] Kaiser, Ann. d. Phys. (4), 13, p. 210; 1904.
[24] Gruschke, S. A. Jahresber, Schles. Ges. f. Vaterl Kultur, Natura, Sept., 1910.
[25] Ahrberg, Diss. Halle; 1909.
[26] Koch, Nova Acta Soc. Upsal (4), 2; 1909.

relative to that of sodium by observing the differences in pressure required to produce successive coincidences among the fringes due to these different colors. These are the earliest measurements of the dispersive power of air; that is, of refractive indices of air for different wave lengths. Lorenz repeated Ketteler's observations on sodium and lithium light. Mascart divided a beam of white light into two parts, which were sent through two parallel tubes filled with gas and then recombined so as to pass through a spectroscope. When the relative pressures in the two tubes were changed, Talbot bands appeared in the spectrum because of the change in optical path. The index of refraction was determined from a count of the number of bands which passed with a definite change in pressure at points in the spectrum corresponding to four lines in the cadmium arc. Koch used an interference refractometer to measure the indices of air for the residual rays from calcite and gypsum. These waves are longer than those corresponding to an infra-red absorption band due to atmospheric carbon dioxide.

The principle of the Jamin refractometer was used by a number of investigators to find the relation of refractive index to density as affected by pressure or temperature variations.

The experiments of Chappuis and Riviere on air at 21° C indicated a proportionality of refractivity to density over a pressure range from 0 to 20 atmospheres. The same conclusion was arrived at by Perreau who worked with air at 16° C up to about 5 atmospheres pressure. Walker measured the refractivity of air under atmospheric pressure at temperatures of 10 to 100° C and found the coefficient of index variation with temperature to be about 2 per cent less than the expansion coefficient. Gale's experiments showed that the proportionality of refractivity and density held to 0.1 per cent for air under pressures from 1 to 20 atmospheres. Magri investigated the refractivity of air under pressures up to 200 atmospheres and preferred the relation $\frac{n^2-1}{(n^2+1)d}=\text{Const.}$ The more simple relation $\frac{n-1}{d}=\text{Const.}$ was again found by Kaiser for air pressures from 0 to 1 atmosphere.

The Fizeau dilatometer is another type of interferometer which has been used for measurements of the index of refraction of air by Benoit [27] and by Scheel.[28] In Benoit's experiments a tripod

[27] Benoit, Jour. de Phys. (2), 8, p. 451; 1889.
[28] Scheel, Verh. d. Deutsch. Phys. Geo., 9, p. 24; 1907.

separated two interferometer plates which were illuminated with sodium light to show fringes. The displacement of these fringes when the temperature of the air between the interferometer plates was changed through the range 0 to 80° C showed the coefficient of index variation with temperature to be identical with the coefficient of expansion of the gas, that is, 0.00367. Scheel measured refractive indices for various wave lengths when the air was at temperatures of 16 and −192° C.

Rentschler [29] was the first to make use of the Fabry and Perot type of interferometer for the measurement of refractive indices of a gas for different light waves. Silvered plates separated 9.005 mm were used with a concave grating. The interferometer was illuminated with a quartz mercury lamp; the systems of circular interference fringes produced by different radiations were separated by the diffraction grating and photographed when a vacuum existed between the interferometer plates and also when various pressures of air were present. The air had a temperature of 21.1° C, but the fringe shifts from which the indices of refraction were computed were corrected for air at 0° C and 760 mm.

The Fabry and Perot interferometer was also used by Miss Matthews [30] and by Miss Howell.[31] Miss Matthews undertook to test the relation of refractive index to density of air but her results are too meager to warrant any conclusion. Miss Howell used a quartz spectrograph with quartz interferometer plates covered with thin nickel films and measured the index of air for 11 points between 2652 A and 6678 A.

4. DIFFRACTION GRATING METHOD

A diffraction grating in a chamber which could be evacuated was used by Runge [32] for dispersion measurements in the extreme ultra-violet and very recently by Dickey [33] for refractive index measurements through a range of spectrum from 2493 A to 6411 A. Dickey placed a bell jar with a quartz window over the grating and determined the ratios of iron arc wave lengths in air and vacuum by measuring the change in their diffraction angles when the air was removed from the jar. Values of the index of refraction were deduced for 28 points in the above-mentioned spectrum range, the value for each point being determined from the measured shifts of an average of 8 lines in the

[29] Rentschler, Astroph. Jl., 28, p. 345; 1908.
[30] Matthews, Jour. Frank. Inst., 177, p. 673; 1914.
[31] Howell, Phys. Rev., 6, p. 81, 1915.
[32] Runge, Wied Ann., 55, p. 44; 1895.
[33] Dickey, Astroph. Jl., 45, p. 188; 1917.

neighborhood. The dispersion was assumed to be linear for this reduction except in the ultra-violet where Kayser and Runge's dispersion formula was used. Great difficulty was encountered with temperature changes in the grating when the space around it was evacuated.

5. SUMMARY OF PREVIOUS WORK.

This brief review of the previous work on the index of refraction and dispersion of air may be summarized as follows: Table 1 is a collection of all the indices of refraction of air which have been given by the observers mentioned above. The last column contains values computed with the aid of a dispersion formula derived from observations which will be described later in this paper.

TABLE 1.—$(n-1)10^7$

	Observer and date							
	Delambre, 1805	Biot and Arago, 1806	Dulong, 1826	Jamin, 1857	Ketteler, 1865	Mascart, 1874	Lorenz, 1880	Bureau Standards, 1918
λ								
4800						2953. 0		2941. 1
5085						2944. 0		2933. 4
5345					2956. 69			2927. 3
5378						2938. 0		2926. 6
5560	2940. 47	2945. 86	2940. 0	2940. 0				2923. 2
5893					2947. 04	2927. 0	2910. 8	2917. 6
6438						2921. 0		2910. 4
6707					2926. 69		2900. 9	2907. 5
	Chappuis and Riviere, 1888	Benoit, 1889	Kayser and Runge, 1893	Perreau, 1896	Walker, 1903	Magri, 1905	Scheel, 1907	Bureau Standards, 1918
2360			3216. 0					3238. 2
2550			3155. 0					3171. 2
2850			3091. 0					3098. 1
2860			3085. 0					3096. 2
3250			3033. 0					3036. 7
4200			2964. 0					2964. 0
4358							2954. 0	2956. 9
4430			2952. 0					2954. 1
4677				2953 0				2944. 9
4710							2946. 0	2943. 9
4800				2951. 0				2941. 1
4922							2937. 0	2937 6
5016							2936. 0	2935. 0
5048							2936. 0	2934. 2
5085				2942. 0				2933. 4
5378				2935. 0				2926. 6
5460						2929. 0	2924. 0	2925. 0
5630			2924. 0					2921. 9
5780							2918. 0	2919. 3
5893	2919. 0	2922. 5		2926. 0	2928. 0			2917. 6
6152							2912. 0	2913. 8
6438				2918 0				2910. 4
7056							2904. 0	2904. 2

TABLE 1.—$(n-1)10^7$—Continued

λ	Rentschler, 1908	Ahrberg, 1909	Koch, 1909	Gruschke, 1910	Cuthbertson, 1910	Howell, 1915	Dickey, 1917	Bureau Standards, 1918
2493							3222.0	3189.3
2567							3182.0	3166.3
2614							3159.0	3152.9
2652						3140.0		3142.8
2665							3138.0	3139.4
2728							3119.0	3124.0
2894						3090.2		3089.7
2967						3073.7		3076.9
3022						3066.2		3067.9
3038							3061.0	3065.3
3188						3043.0		3043.2
3293							3030.0	3031.4
3341	3036.0							3026.0
3450							3012.0	3015.0
3576							3003.0	3003.5
3650	2997.0						2997.0	2997.7
3867							2981.0	2982.3
3941							2977.0	2977.8
3965						2980.0		2975.5
4046	2972.0							2971.9
4119							2967.0	2967.9
4220							2965.0	2963.1
4315							2959.0	2958.7
4358	2956.0	2957.0	2970.9					2956.9
4409							2955.0	2954.9
4471				2978.0		2955.5		2952.3
4518							2951.0	2950.9
4619							2945.0	2947.0
4712				2969.0		2944.0	2942.0	2943.9
4861					2951.1			2939.3
4879							2940.0	2938.7
4922				2962.0				2937.6
5016				2960.0		2935.0		2935.0
5140							2929.0	2932.1
5202							2926.0	2930.5
5270							2925.0	2928.9
5328							2925.0	2927.7
5406							2922.0	2926.1
5460	2930.0	2925.0	2937.0		2936.0			2925.0
5616							2919.0	2922.1
5769	2925.0							2919.4
5780		2919.						2919.3
5790					2929.8			2919.2
5876						2919.6		2917.8
5893			2929.8	2939.0				2917.6
6231							2914.0	2912.8
6412							2910.0	2910.6
6438		2911.0						2910.4
6563					2919.2			2908.9
6678				2930.0		2912.0		2907.7
6766			2918.5					2907.5
67094			2880.6					
86784			2887.5					

With a few exceptions the results in Table 1 are given by the various observers to represent indices of refraction of air at 0° C and 760 mm. In most cases, however, the measurements were not made with air actually at 0° C and 760 mm, but at various temperatures and pressures and then corrected for the density of the air. In some cases water vapor was present in the air, and a few observers have neglected to state the temperature,

pressure, or constitution of the air with which they worked. Although these results are for these reasons not strictly comparable, their disagreement on the whole does not appear to be very large, considering the fact that they represent such a large range of experimental methods, apparatus, and observers. Most of the dispersion measurements, however, have been confined to a comparatively small spectral region, and dispersion formulas given to represent these measurements disagree by more than 10 per cent of the refractivity $(n-1)$ of air for ultra-violet light.

The Cauchy dispersion formulas given by various observers to represent the refractivity of air (0° C and 760 mm) as a function of the wave length in Angstroms are collected here for purposes of comparison, together with some of the values of $(n-1)\,10^7$ resulting from the formulas.

Dispersion Formulas for Dry Air (0° C and 760 mm)

Kayser and Runge....$(n-1)10^7 = 2881.7 + \frac{13.16}{\lambda^2 \times 10^{-8}} + \frac{0.316}{\lambda^4 \times 10^{-16}}$

Rentschler.....$(n-1)10^7 = 2903.1 + \frac{3.80}{\lambda^2 \times 10^{-8}} + \frac{1.23}{\lambda^4 \times 10^{-16}}$

Cuthbertson.....$(n-1)10^7 = 2885.4 + \frac{13.38}{\lambda^2 \times 10^{-8}} + \frac{0.50}{\lambda^4 \times 10^{-16}}$

Howell.....$(n-1)10^7 = 2881.7 + \frac{11.83}{\lambda^2 \times 10^{-8}} + \frac{0.50}{\lambda^4 \times 10^{-16}}$

Dickey.....$(n-1)10^7 = 2878.12 + \frac{11.59}{\lambda^2 \times 10^{-8}} + \frac{0.551}{\lambda^4 \times 10^{-16}}$

Bureau of Standards..$(n-1)10^7 = 2875.66 + \frac{13.412}{\lambda^2 \times 10^{-8}} + \frac{0.3777}{\lambda^4 \times 10^{-16}}$

Observer	λ2000	λ4000	λ6000	λ8000
Kayser and Runge	3408.2	2976.3	2920.7	2903.0
Rentschler	3766.9	2974.9	2923.1	2912.0
Cuthbertson	3532.4	2988.6	2926.4	2907.5
Howell	3464.9	2973.6	2918.1	2901.3
Dickey	3512.2	2972.1	2914.6	2897.6
Bureau of Standards	3447.1	2973.2	2915.8	2897.5

Four different expressions have been proposed to represent the relation between the index of refraction of a substance and its density. The earliest one was supported by Newton and indicated $\frac{n^2-1}{d} = \text{constant}$. An empirical relation $\frac{n-1}{d} = \text{constant}$ was found by Dale and Gladstone [34] in measurements of refractive indices of liquids at various temperatures. A new relation $\frac{n^2-1}{(n^2+2)d} = \text{constant}$, was deduced by Lorenz [35] from the elastic

[34] Dale and Gladsbone, Phil. Trans., 151, p. 317: 1863. [35] Lorenz, Wied Ann. 11, p. 70; 1880.

solid theory and later from the electron theory by Lorentz.[36] The fourth relation, $\frac{(n^2+1)d}{n^2-1}$ = constant, was proposed by Magri[37] to represent his measurements on air under high pressure. Experiments on gases form the best test of these various relations since the density of gases can be varied more easily than that of solids and liquids. Many of the investigations on air mentioned above were undertaken to find the exact relation of index of refraction to density, but the results are more or less in conflict and point to no definite choice between the different relations proposed. It is evident that this choice depends on extremely accurate measurements, since n^2-1 differs from an integral multiple of $n-1$ only by a term of the order $(n-1)^2$. For air $n-1=0.0003$, approximately, so that $(n-1)^2=9\times10^{-8}$. It may be that changes in pressures or temperature used to change the density of the air introduce errors in the observations which are much larger than the difference between the relations under test so that as far as can be judged from measurements on air the proposed relationships of index and density are indistinguishable.

II. PURPOSE

1. WAVE LENGTH CORRECTIONS FOR TEMPERATURE AND PRESSURE OF AIR

The measurements of indices of refraction of air which are described below were inspired by spectroscopic problems connected with the accurate measurement of wave lengths. The fundamental spectroscopic standard has been defined by the International Wave Length Committee as 6438.4696 A which represents the length of a wave of Cadmium red radiation in dry air at 15° C and 76 cm. Other radiations whose wave lengths are determined relative to the primary standard are called secondary standards and a large number of these are required throughout the entire spectral range to facilitate spectroscopic work. When determinations of secondary standards are made in air at temperatures other than 15° C, and pressures differing from 76 cm. the determinations require corrections which demand a knowledge of the dispersion of the air and the dependence of indices of refraction on density.

[36] Lorentz, Theory of Electrons, p. 143. [37] Magri, Phys. Zeitschr., 6, p. 629; 1905.

2. WAVE LENGTH CORRECTIONS TO VACUUM

Furthermore, it is very desirable to be able to convert wave lengths measured in air to their values in a vacuum. This is especially important for the discussion of mathematical relations among spectral lines in which it is best to use the vacuum values of oscillation frequencies. For this purpose oscillation frequency is defined as the number of waves contained in 1 cm in vacuum; that is, the reciprocal of the wave length in centimeters reduced to its value in vacuum. To reduce the wave length of any line measured in air to its real value in vacuum it is necessary to multiply by the refractive index of air for light of the particular wave length. For air at 15° C and 76 cm this correction amounts to +2.468 A for the wave length 9000 A or −3.05 in the oscillation frequency. At 2000 A the correction to the wave length is +0.651 A or −16.28 to the oscillation frequency. In the early work on spectral series the accuracy of the wave length measurements was not very great and did not warrant correction to vacuum value.

For this reason Kayser and Runge used oscillation frequencies deduced from wave lengths measured in air without reduction to vacuum. Recently, the accuracy of wave length measurements has been increased by the use of the interferometer for wave-length comparisons, and also by the use of large diffraction gratings working with the new system of international standards of wave length derived from interferometer experiments. An accuracy of one part in several millions is now striven for in wave length measurements and to maintain this relative accuracy in the values converted to vacuum it is necessary to know the indices of refraction to two or three units in the seventh decimal place.

This Bureau has been interested in the accurate measurement of wave lengths in spectra of the chemical elements and has had considerable success in extending such measurements out into the infra-red region of the spectrum. Interferometer comparisons have been made in the iron spectrum to 9000 A[38] and in the spectra of neon and argon to 8500 A.[38] Grating spectra have been photographed and measured in many cases beyond 9000 A [39] and waves longer than 10 000 A have been measured in the arc spectra of the ferrous metals.[38]

To correct these measurements to standard atmospheric conditions, and also to change them to vacuum values, requires extensive and accurate data on the refractive and dispersive powers

[38] Unpublished. [39] This Bulletin, 14, p. 371; 1917.

of air. The introductory review and summary of such data show that although a large number of researches on the index of refraction of air have been made, employing many different experimental methods, the observations up to the present time have covered a comparatively small spectral region and are often in rather wide disagreement. Most observers have been content with index determinations for one point, or at most only a few points, in the ultra-violet and short wave visible spectral regions, and no values of the index of air have ever been published for waves in the extreme red and adjacent infra-red regions.

The work described in the present paper was first undertaken for the purpose of measuring the indices of refraction of air for red and infra-red light. When this was completed the observations were carried on through the visible spectrum and pushed out into the ultra-violet until practically the entire spectral region which is accessible to ordinary photography was investigated.

III. EXPERIMENTAL METHOD

1. FABRY AND PEROT INTERFEROMETER FOR MEASUREMENT OF n

For the measurement of absolute indices of refraction of air, convenience and efficiency together with greatest accuracy recommended the method which makes use of the Fabry and Perot interferometer. This apparatus allows the most satisfactory control of temperature and pressure of the air whose optical properties it is desired to determine. It also permits observations to be recorded simultaneously by photography for a large number of different wave lengths. These advantages determined the use of the Fabry and Perot type of interferometer for the present investigations, and a detailed description of the apparatus will be given in Section IV of this paper.

The experimental method was as follows: Diametric sections of the circular (Haidinger) rings produced by the various rays from a source of light illuminating the parallel plates of the interferometer were photographed, first, when a vacuum existed between the interferometer plates, and then when dry air at atmospheric pressure was present. If the distance between the interferometer plates is e, then the number of waves p (order of interference) of length λ existing in this distance when air is absent is $p = e/\lambda$. When dry air at atmospheric pressure is present between the plates the wave length λ is shortened to λ', and consequently the number of waves contained in the same distance

e is increased to $p'=e/\lambda'$. The total order of interference p at the center of the system of circular fringes in general consists of a whole number P and a fraction p_1. The fraction p_1 is determined from measurements of the ring diameters and an approximate value of P. If these fractions are obtained for three or more different but known wave lengths, the whole number P and finally the distance e can be found readily and accurately by a method previously described.[41] Similar procedure is required for the determination of p^1 and P' when air exists between the interferometer plates. When the wave-length values in air were used for finding the order of interference for vacuum between the plates, differences were observed between the computed and measured fractional parts of the order because of the dispersion of the air. These differences were not large enough, however, to introduce any difficulty or ambiguity in the determination of the correct order of interference.

By definition the index of refraction n of air for wave length λ, is $n=\frac{\text{velocity of light of wave length } \lambda \text{ in vacuum}}{\text{velocity of light of wave length } \lambda' \text{ in air}}$. In either case the velocity is the product of frequency and wave length; and since frequency is assumed to remain constant we may write $n=\frac{\lambda\ (=\text{wave length in vacuum})}{\lambda'(=\text{wave length in air})}$. Now apply the interferometer to the measurement of n. When the space between the interferometer plates is exhausted of air, the distance e is found to contain p waves of length λ. When air at a definite temperature and pressure is present between the plates, p' waves of length λ' are found to be contained in e. Since e is a constant distance, $e=p\lambda=p'\lambda'$, or $\lambda/\lambda'=p'/p$, and the index of refraction of air under the conditions in which p' is observed can be obtained directly from the observed orders of interference p and p', that is,

$$n=\frac{p'(=\text{number of waves in } e \text{ air})}{p\ (=\text{number of waves in } e \text{ vacuum})}.$$

IV. DESCRIPTION OF APPARATUS

1. GENERAL PLAN

The general plan of the apparatus is represented in Figs. 1 and 2, which differ only in the dispersive apparatus.

The image of a source of light O is projected with threefold magnification by a quartz lens L, upon the interferometer plates

[41] This Bulletin, 12, p. 198; 1915-1916.

I, which are inclosed in a cylinder A, whose ends are closed with quartz windows. An image of the circular fringes produced by the interferometer is projected upon the slit S of a spectrograph by means of a quartz fluorite achromatic objective L_2. For most of the work a grating spectrograph was used with the grating mounted in parallel light as shown in Fig. 1, where M represents a speculum concave mirror, G the concave grating, and P the photographic plate. The mirror and grating each have a radius of curvature of about 640 cm. The grating was ruled by Dr.

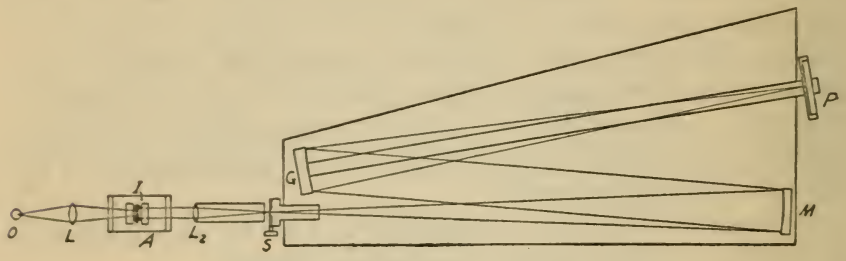

FIG. 1.—*Grating spectrograph*

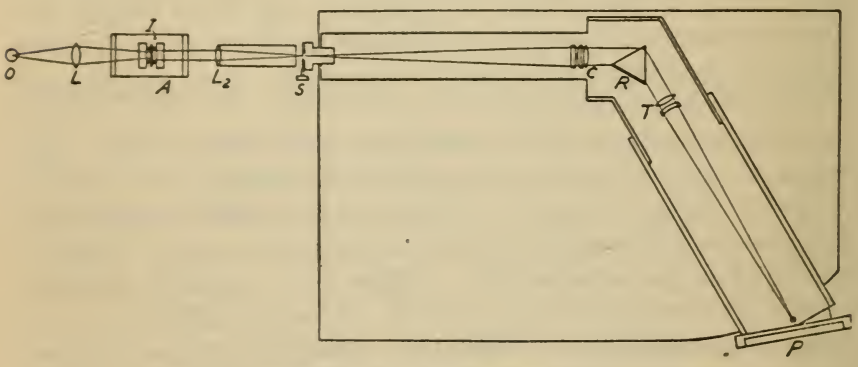

FIG. 2.—*Prism spectrograph*

Anderson and possesses 39 800 lines on a distance of 13.2 cm. The linear dispersion at P with this apparatus is about 10 A per millimeter in the spectrum of the first order.

In the region of the spectrum where the waves are shorter than 2800 A the reflecting power of speculum metal becomes very small and too much light was lost in the reflections from the mirror and grating. For these short waves, therefore, a prism spectograph whose optical parts consist of quartz and rock salt was used to separate the different radiations emanating from the source of light. The prism R, Fig. 2, was of rock salt with a 60° angle and

3-inch refracting edge. The collimator C is a quartz rock salt objective and the camera lens T is made of quartz. This spectrograph was designed for the photography of interference fringes in the spectral region 2000–3000 A, and gives a linear dispersion of about 1 A per millimeter (at $\lambda = 2500$ A) in the focal plane where the photographic plate P is placed.

2. SOURCES

The sources of light used in this work were chosen by the following qualifications: (1) The spectra must show a large number of lines of sufficient intensity and regularity of distribution. (2) The lines must be sufficiently sharp or homogeneous to show good interference fringes with comparatively large path differences or orders of interference.

The standard iron arc was used through the range of wave lengths from 2750 A to 9000 A and interferometers were used which made the average order of interference about 20 000 waves. In the long wave region these observations were supplemented by the use of electrical discharge tubes containing neon and argon. Observations were made with the spectrum of neon between the wave-length limits 5852 A and 8495 A, and with the spectrum of argon from 6500 A to 8521 A. The radiations from these gases are quite homogeneous and they were therefore used with a larger order of interference which averaged about 70 000 waves. In the extreme ultra-violet region the most satisfactory source was found to be a copper arc with a positive carbon electrode above. This spectrum was used for observations in the wave length interval 2200 A–3200 A with orders of interference averaging 16 000 waves.

3. INTERFEROMETERS

The interferometer plates were glass or quartz disks of 42 mm diameter and 6 or 8 mm thickness. These plates were thinly covered with metallic films by the method of sputtering from a metallic cathode in a vacuum. Four different pairs of plates were used in all, each pair being used in the spectral region where its metallic films showed their highest reflecting power and the least absorption. In the red and infra-red parts of the spectrum glass plates with thin copper films were used. In the visible spectrum glass plates with silver films were used. For the short visible waves and adjacent ultra-violet quartz plates with nickel films were found most efficient while silicon films on quartz plates were found greatly superior to nickel in the shortest ultra-violet.

The interferometer plates *I*, Fig. 3, were held apart and parallel by three invar posts of equal length, held in an invar ring. These invar separators or etalons ranged in length from 2 to 25 mm depending on the source of light and the spectral region which was being used. The system of plates and etalons was placed in a hollow cylinder (5 cm by 10 cm), and held against a flange on one end of this cylinder by screwing a disk *D* down the other end. This disk had a 2 cm hole in its center to allow the passage of light through the interferometer. Three small screws in the flange pressed against springs which bore on the interferometer plate facing the flange and permitted the adjustment of the plates to parallelism by varying the pressure on the separating post.

The disk screw *D* allowed the adaptation of this interferometer mounting to separators or etalons ranging from 2 to 50 mm in length. After the interferometer plates were adjusted so as to be

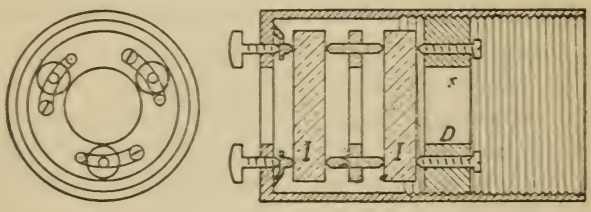

FIG. 3.—*Interferometer*

strictly parallel, the entire interferometer mounting was slipped into a closely fitting brass tube 40 cm long which was surrounded by a bath whose temperature could be regulated at will as described below.

4. AIR CHAMBER

In order to secure an accuracy in $n-1$ of 1 in the fourth significant figure, it was necessary to control the temperature of the air under investigation and the interferometer to 0.1° C and the pressure of the air to 0.3 mm of mercury. To accomplish this the air chamber *A*, Fig. 4, was surrounded by a bath of alcohol and water. The liquid was circulated through the tubes *D* and *E* by the motor-driven propeller *F*. Cooling was supplied to the liquid by the brine coil *C* and heat by a 5-ohm resistance coil *H*, the current in which was regulated by a relay operated by the thermostat *J*. This apparatus was surrounded with ground-cork insulation and mounted in a wooden box. With this arrangement the liquid around *A* could be held within 0.01° C of the required

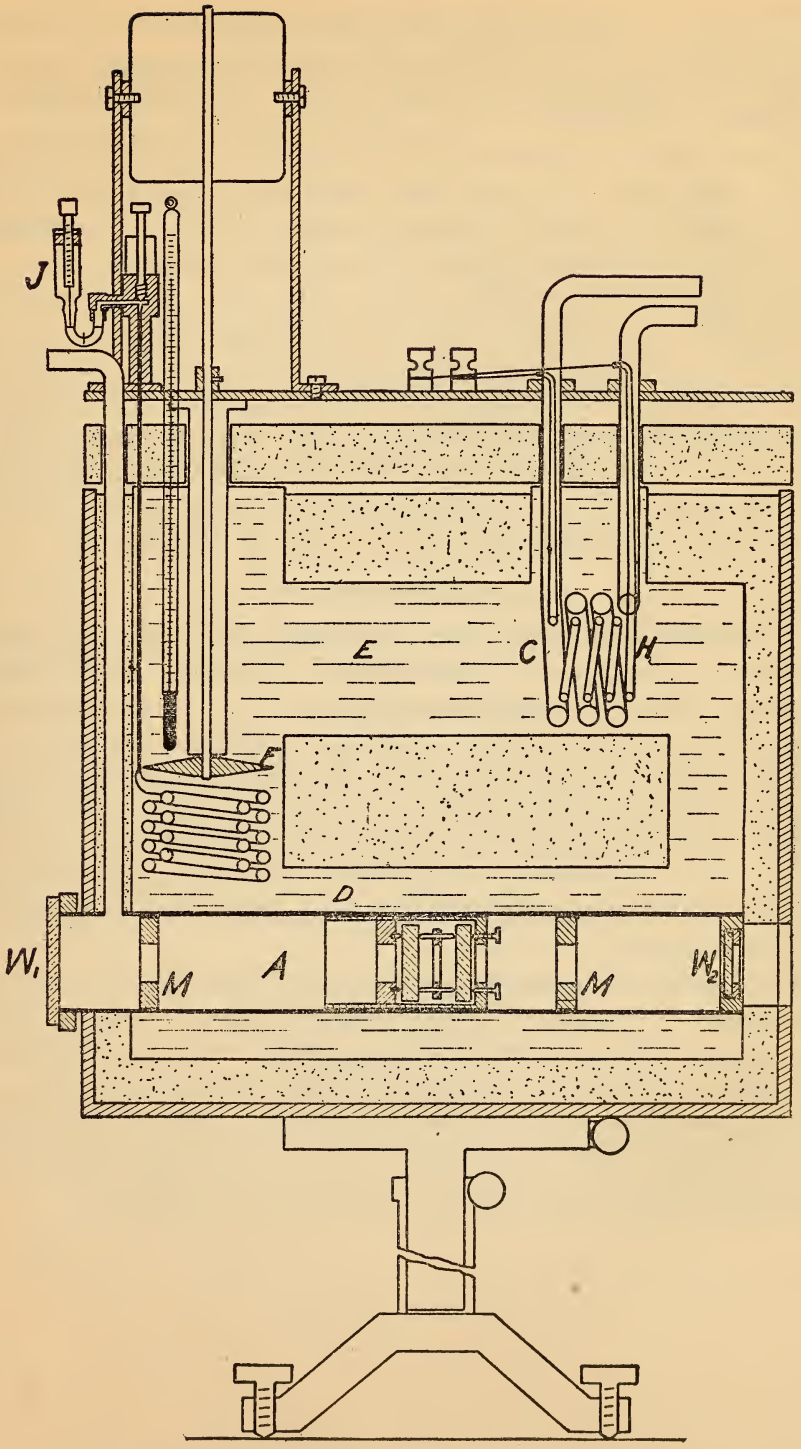

FIG. 4.—*Interferometer in temperature control apparatus*

temperature for any length of time. Temperatures were read with calibrated thermometers Nos. 8290 C and 14 454 C, and a 5-junction copper-constantin thermocouple calibrated by the heat division of this Bureau.

The chamber A consisted of a thin brass tube 40 cm long and 5 cm diameter. The ends were closed with brass rings having 2 cm openings covered with quartz windows, W_1W_2. The cylinder containing the interferometer I fitted closely within the tube A, thus making good metallic contact with the bath. Two brass rings, MM, 1 cm thick, reduced the radiation from the ends of the tube. A temperature survey was made of A with differential thermocouples when the bath was 20° above the room temperature. The interferometer plates remained within 0.01° of constant temperature for one-half hour and differed from the bath temperature by less than 0.02° C. The plates were 0.02° warmer than M, and 0.7° above the temperature of the ends of the tube A.

5. PRESSURE GAGES

The air was exhausted from the cylinder containing the interferometer by means of a Gaede rotary oil pump. An electrical discharge tube inserted between the pump and the cylinder and operated with a 10 000-volt transformer indicated the degree of exhaustion in the apparatus. For the vacuum observations the apparatus was exhausted of air until the dark space around the electrodes of the electrical discharge tube reached from 5 to 10 mm. This low pressure, corresponding to a few hundredths of a millimeter of mercury was maintained throughout the photographic exposure period.

The pressure observations were made after allowing air at atmospheric pressure to filter through calcium chloride and phosphorus pentoxide and fill the apparatus. These atmospheric pressures were usually read from two different barometers used as working standards in this Bureau, and all the necessary corrections were carefully made so that the final pressure observations are probably correct to one or two hundredths of a millimeter.

6. CONSTITUTION OF THE AIR

The barometer readings represent the pressure of dry air whose constituents at the earth's surface are considered to be present in the following volume percentages:[42]

Nitrogen	78.03
Oxygen	20.99
Argon	.94
Hydrogen	.01
Neon	.0012
Helium	.0004
Carbon dioxide	.03

Krypton, Xenon, and Ozone in very small quantities. When the air was removed from the apparatus the spectrum from the electrical discharge tube showed only a trace of hydrogen remaining.

Next to water vapor the atmospheric constituent in which the largest variation occurs is carbon dioxide. The laboratory in which these experiments were conducted was quite well ventilated, so that the volume percentage of carbon dioxide could not have exceeded two times the normal amount.

7. PHOTOGRAPHIC

The short wave spectral region from the blue to the extreme ultra-violet was recorded on Graflex photographic plates. For longer waves from green to red in the visible spectrum Seed 27 plates were used after staining with pinacyanol. Overlapping spectra were absorbed by a cell of potassium-bichromate solution. The longest waves in the red and adjacent infra-red were photographed on Seed 27 plates after staining with dicyanin. Here the overlapping spectra were removed by placing a screen of Jena red glass No. 4512 in front of the slit of the spectrograph.

The photographic plates measured 5 by 8 inches. Each plate therefore covered an interval of 2000 A with the grating spectrograph and about 1000 A with the prism spectrograph in the extreme ultra-violet. The cameras were built so as to allow vertical displacement of the plate holders for a series of exposures on the same plate.

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[42] Hann. Lehrbuch der Meterologie, 3d ed., p. 5.

V. EXPERIMENTAL PROCEDURE

1. OPTICAL ADJUSTMENTS

After the interferometer plates were made parallel to each other the interferometer mounting was inserted to the middle of the cylinder surrounded by the constant temperature bath. The box containing this apparatus stood on a tripod fitted with leveling screws and rotating top by means of which the interferometer could be leveled and oriented so that the center of the circular fringe system fell exactly on the center of the spectrograph slit.

2. TEMPERATURE REGULATIONS

Spectroscopists are concerned with the index of refraction and dispersion of air at atmospheric pressures and room temperatures since most wave length measurements are made in air under these conditions of pressure and temperature. In these experiments, therefore, extensive observations were made on these optical properties of air at atmospheric pressures and at temperatures of 0, 15, and 30° C, which cover the range of ordinary working conditions.

The temperature of the bath surrounding the interferometer was brought to the desired point and maintained by the thermostat control throughout the experiment. This temperature was held for a period of one to two hours before beginning the photographic exposures in order to eliminate temperature gradients and establish a steady state in the interferometer. Cooling or heating effects due to letting air in or exhausting the air from the apparatus were guarded against by allowing 10 to 15 minutes to elapse after each of these operations for a return to the steady state.

3. PRESSURE OBSERVATIONS

Each set of photographic exposures was begun with a vacuum between the interferometer plates. The apparatus was washed out with dry air once or twice before the low pressure was maintained for the first exposure. This low pressure, as indicated by the electrical discharge tube, corresponded in general to a pressure of only a few hundredths of a millimeter of mercury. After the observations for vacuum between the interferometer plates were recorded a two-way stopcock cut off the pump and admitted air from the room after being dried by passing through calcium chloride and phosphorus pentoxide. The pressure of this air was read from the barometers described above. When the exposures

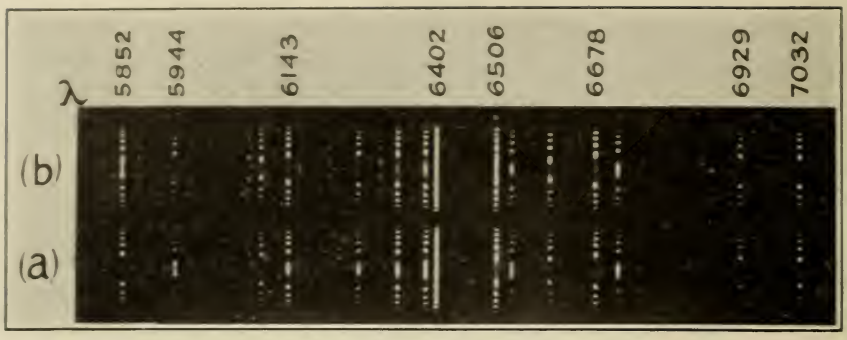


FIG. 5.—*Interference fringes in the spectrum of neon*

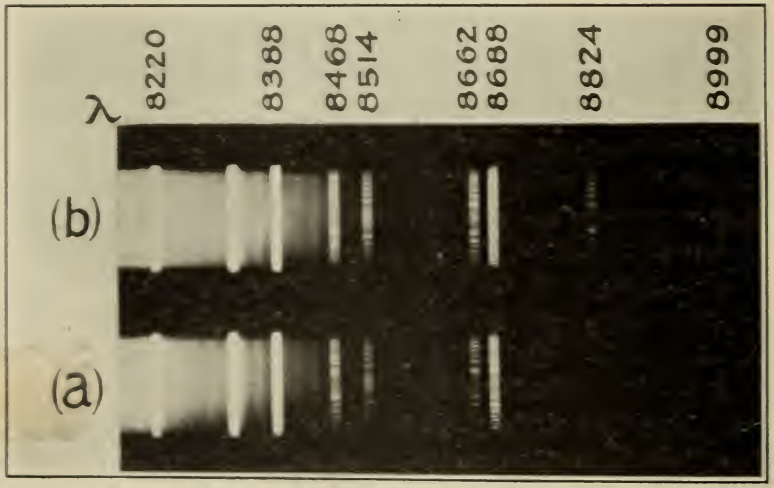


FIG. 6.—*Interference fringes in the spectrum of iron*

were longer than about 5 minutes the pressure observations were made before and after and the mean value was used in the density reductions.

4. PHOTOGRAPHIC EXPOSURES

In general five photographic exposures were made for each experiment, from which the indices of refraction of air at a definite density could be determined for a certain wave length region. These exposures were made in the following order: (1) Interference rings for vacuum between the interferometer plates; (2) interference rings for dry air between the interferometer plates; (3) interference rings for vacuum between the interferometer plates; (4) interference rings for dry air between the interferometer plates; and (5) gages from which the diameters of the photographed rings could be transformed to their actual size as projected on the slit of the spectrograph. These five exposures were generally made on a 5 by 8 plate, the plate holder being lowered about 1.5 cm between exposures.

The exposure times in the visible and adjacent ultra-violet spectrum ranged from 1 to 10 minutes. The longest infra-red and shortest ultra-violet waves required exposures of 60 minutes. Spectrum regions were chosen in successive experiments to overlap half of the preceding region.

Several photographs which are typical of those made in this work are reproduced in Figs. 5 and 6. Fig. 5 shows the yellow and red portion of the spectrum of neon with interference maxima and minima in the spectrum lines. These fringes were produced by an interferometer whose silvered plates were separated 3.75 mm. The first exposure (*a*) was made after the air was removed, and the second (*b*) was made with dry air at atmospheric pressure and 15° C between the interferometer plates. Fig. 6 shows similar interference phenomena in the infra-red spectrum of iron. These fringes were produced by a 10 mm separation of the interferometer plates with copper films. Wave lengths in dry air at atmospheric pressure and 30° C caused (*a*), and wave lengths in a vacuum gave (*b*).

5. MEASUREMENT OF RING DIAMETERS

The diameters of the interference rings were measured by means of a micrometer screw with a head graduated to 5μ. The diameters of the first three rings were measured for most of the lines which were used, and for the remainder the first two rings were

measured. In general, these measurements made it possible to compute the orders of interference within a few thousandths of a wave. The fractional parts of the orders of interference agreed so well for the two vacuum and the two air exposures in each experiment that the means of the two sets were used for index computations in every case. No appreciable systematic differences were observed even in cases where the barometric pressure changed slightly between the two air exposures.

VI. REDUCTION OF OBSERVATIONS

1. CORRECTIONS TO 760 MM

The index of refraction n of air at a measured density was deduced for a particular wave length λ by dividing the number $(p' = P' + p'_1)$ of waves in the double distance $2e$, separating the interferometer plates when dry air of determined density was present by the number $(p = P + p_1)$ in this same distance when only a negligible quantity of air remained. The number of waves in $2e$ air was from 5 to 25 greater than the number in $2e$ vacuum, depending on the size of the etalon and the wave length.

The density of the air experimented upon depended on its temperature t and the barometric pressure B observed at the time of the experiment. The observed barometric pressures ranged between 730 and 760 mm. The final values of n were deduced for dry air under the standard pressure of 760 mm of Hg, by multiplying the value n_B obtained from the observed barometric pressure B by $\frac{760}{B}$; that is, $n_{760} = n_B \frac{760}{B}$. This assumes that the coefficient of variation of the index of refraction with density is identical with the pressure coefficient of volume change at constant temperature. Although considerable disagreement exists among previous observers as to the exact relation between refractivity and density of air, the deviation from exact proportionality is too small to be appreciable here since the barometric pressures never deviated more than 30 mm from 760 mm in these experiments.

2. FINAL VALUES OF n—1

All of the observations are collected in Table 2. Each value of $n - 1$ is the mean of the number of observations indicated in the next column, and this is followed by the correction necessary to place this value on the dispersion curve whose equation was determined from all the observations.

TABLE 2.—(n—1) 10^7 for Dry Air

λA	Source	Observations, 0° C, 760 mm	Number of observations	Computed minus observed	Observations, 15° C, 760 mm	Number of observations	Computed minus observed	Observations, 30° C, 760 mm	Number of observations	Computed minus observed
2218	Cu	.....	.....	.....	.....	.....	.....	2982.7	1	−37.3
2246	Cu	.....	.....	.....	.....	.....	.....	2960.4	2	−26.4
2303	Cu	.....	.....	.....	.....	.....	.....	2907.5	2	+ 4.9
2369	Cu	.....	.....	.....	.....	.....	.....	2879.0	2	+10.9
2392	Cu	3223.4	2	− 1.6	3050.0	2	− 0.5	2867.1	2	+15.5
2406	Cu	3222.7	1	− 2.8	.....	.....	.....	2869.4	2	+ 9.0
2441	Cu	3209.0	2	− 1.6	3046.6	2	−14.0	2862.3	2	+ 5.2
2492	Cu	3196.6	2	− 7.1	3012.2	2	+ 4.2	2857.7	2	− 3.8
2618	Cu	3145.8	2	+ 6.4	2962.4	2	+19.4	2833.6	2	−10.0
2739	Fe	3101.7	1	+11.0	2932.0	1	+22.4	2804.0	1	− 4.0
2766	Cu	3139.4	1	−23.4	2950.5	2	− 2.2	2787.1	2	+ 7.4
2824	Cu	3103.8	2	− .3	2940.8	2	− 4.2	2779.3	2	+ 4.8
2851	Fe	3085.8	1	+ 2.1	.....	.....	.....	2768.3	2	+11.3
2882	Cu	3080.6	1	+11.3	2921.0	2	+ 4.9	2774.1	1	+ .6
2918	Fe	3079.8	1	+ 5.4	2912.3	1	+ 7.3	2752.1	2	+17.1
2961	Cu	3078.3	2	− .6	2914.0	2	− 1.2	2746.2	1	+16.6
2987	Fe	3073.7	1	− .3	2918.0	1	− 9.2	2744.8	1	+14.4
2997	Cu	3074.2	2	− 2.5	2915.3	1	− 8.0	2742.6	2	+15.3
3010	Cu	3078.2	2	− 8.6	2894.8	1	+10.5	2751.9	2	+ 4.4
3036	Cu	3084.1	1	−18.5	2908.2	1	− 6.6	2745.0	2	+ 7.8
3055	Fe	.....	.....	.....	2910.5	1	− 1.7	.....	.....	.....
3063	Cu	3066.1	2	− 4.6	2901.2	1	− 3.5	2750.7	1	− 1.0
3075	Fe	.....	.....	.....	.....	.....	.....	2747.3	1	+ .9
3116	Fe	3048.9	1	+ 5.0	2889.8	1	+ .8	2746.4	1	− 3.1
3175	Fe	3034.3	1	+11.5	2894.0	1	−10.7	2720.2	1	+16.6
3205	Fe	3038.2	1	+ 3.8	2884.9	1	− 5.2	2727.1	1	+ 6.2
3280	Fe	3028.9	1	+ 4.0	2872.5	1	− 1.1	2732.5	1	− 6.6
3347	Fe	3022.7	1	+ 2.8	2857.0	1	+ 7.3	2720.8	1	− 1.2
3413	Fe	3023.6	1	− 5.0	2859.7	1	− 1.5	2715.6	1	− 1.7
3485	Fe	3004.5	1	+ 7.2	2857.0	1	− 5.1	.....	.....	.....
3513	Fe	3010.0	1	− .9	2858.0	1	− 8.5	2716.0	1	−10.1
3594	Fe	2988.9	1	+13.2	2845.0	1	− 2.1	2714.9	1	−14.5
3640	Fe	2986.1	1	+12.3	2842.4	1	− 3.0	2700.0	1	− 2.9
3659	Fe	2998.7	3	− 1.8	2836.3	3	+ 1.9	2699.6	3	− 3.8
3701	Fe	2996.4	1	− 2.7	2834.2	1	+ .9	2679.0	1	+13.9
3753	Fe	2988.0	4	+ 1.9	2831.8	4	− .2	2688.0	4	+ 1.6
3787	Fe	.....	.....	.....	2831.6	2	− 2.2	.....	.....	.....
3790	Fe	.....	.....	.....	2829.1	1	+ .1	.....	.....	.....
3843	Fe	2977.5	1	+ 6.3	2829.2	4	− 3.4	2689.5	3	− 4.9
3846	Fe	.....	.....	.....	.....	.....	.....	2682.2	1	+ 1.2
3865	Fe	.....	.....	.....	2826.1	2	− 1.6	.....	1	.....
3867	Fe	2981.4	3	+ .8	.....	.....	.....	.....	.....	.....
3906	Fe	2993.3	1	−13.5	2821.6	3	+ .6	2676.1	1	+ 5.0
3935	Fe	2979.8	3	− 1.8	2814.8	3	+ 5.8	2689.1	3	− 9.5
3969	Fe	.....	.....	.....	2819.8	2	− 1.0	.....	.....	.....
3977	Fe	2966.2	1	+ 9.3	2815.9	1	+ 2.4	2681.4	1	− 4.0
3983	Fe	2973.7	3	+ 1.5	2821.4	3	− 3.4	2677.3	3	− .2
4005	Fe	.....	.....	.....	2817.5	2	− .7	.....	.....	.....
4021	Fe	2971.7	4	+ 1.4	2815.7	4	+ .3	2669.9	4	+ 5.6
4076	Fe	2970.4	1	− .2	2816.4	1	− 3.1	2688.7	1	−15.8
4095	Fe	2966.6	3	+ 2.5	2809.5	3	+ 2.9	2676.0	3	− 4.0
4118	Fe	2973.4	4	− 5.5	2809.3	2	+ 2.0	.....	.....	.....
4147	Fe	.....	.....	.....	2807.8	4	+ 2.1	2666.8	4	+ 2.9
4191	Fe	.....	.....	.....	2807.7	2	+ .2	.....	.....	.....
4210	Fe	2970.0	1	− 6.6	2808.1	1	− 1.1	2668.7	1	− 1.6
4213	Fe	2963.6	3	− .4	2805.3	3	+ 1.6	2669.7	3	− 2.7
4233	Fe	.....	.....	.....	2807.1	2	− 1.1	.....	.....	.....
4245	Fe	2965.9	3	− 4.2	2804.7	3	+ .8	2668.0	3	− 2.3
4282	Fe	2956.1	1	+ 3.9	2807.6	3	− 3.6	2650.7	1	+13.5
4315	Fe	2960.8	3	− 2.2	2800.4	5	+ 2.3	2663.8	3	− .8
4352	Fe	2967.5	1	−10.5	2805.9	1	− 4.7	2662.8	1	− 1.2
4369	Fe	2954.2	3	+ 2.1	2796.1	3	+ 4.5	2663.9	3	− 2.9
4375	Fe	.....	.....	.....	2798.3	2	+ 2.1	.....	.....	.....
4422	Fe	2954.7	3	− .6	2792.8	3	+ 5.7	2664.4	3	− 5.5
4427	Fe	.....	.....	.....	2796.7	2	+ 1.6	.....	.....	.....

TABLE 2—Continued

λA	Source	Observations, 0° C, 760 mm	Number of observations	Computed minus observed	Observations, 15° C, 760 mm	Number of observations	Computed minus observed	Observations, 30° C, 760 mm	Number of observations	Computed minns observed
4484	Fe	2950.9	3	+ 0.9						
4494	Fe				2796.4	5	− 0.4	2656.2	3	+ 0.5
4531	Fe				2794.4	2	+ .3			
4547	Fe	2947.3	3	+ 2.1	2789.0	3	+ 5.2	2655.0	3	.0
4592	Fe	2943.4	3	+ 4.4	2791.9	3	− .2	2654.9	3	− 1.2
4602	Fe				2790.7	2	+ .6			
4647	Fe	2946.3	3	− .4	2787.8	3	+ 3.1	2654.9	3	− 2.8
4691	Fe	2943.0	3	+ 1.4	2785.2	3	+ 4.4	2649.7	3	+ 1.2
4736	Fe	2938.9	3	+ 4.1	2789.2	5	− .9	2653.6	3	− 4.0
4789	Fe	2935.7	3	+ 5.6	2778.4	3	− 1.7	2647.4	3	+ .6
4859	Fe	2938.0	3	+ 1.3	2787.6	5	− 2.8	2646.5	3	− .2
4903	Fe	2937.6	3	+ .4	2783.2	5	+ .5	2647.1	3	− 1.9
4966	Fe	2933.2	3	+ 3.0	2789.1	3	− 7.0	2646.0	3	− 2.3
5001	Fe	2935.5	3	− .2				2642.0	3	+ .8
5012	Fe				2783.0	5	− 2.1			
5051	Fe	2935.9	3	− 1.9	2764.0	3	+16.0	2642.0	3	− .3
5110	Fe	2930.5	5	+ 2.1	2784.3	2	− 5.6	2643.3	5	− 3.0
5167	Fe	2926.8	3	+ 4.4	2779.3	5	− 1.8	2639.0	3	+ .2
5171	Fe	2930.9	2	+ .2	2776.9	2	+ .5	2637.3	2	+ 1.8
5232	Fe	2934.2	5	− 4.4	2779.2	5	− 3.1	2636.4	5	+ 1.5
5266	Fe	2938.7	2	− 9.8				2636.5	2	+ .7
5324	Fe	2936.1	2	− 8.4	2775.9	2	− 1.6	2630.8	2	+ 5.4
5397	Fe	2930.1	2	− 3.9	2771.9	2	+ .9	2636.1	2	− 1.3
5455	Fe	2924.4	2	+ .6	2773.4	2	− 1.7	2634.3	2	− .5
5506	Fe	2922.6	2	+ 1.4	2772.8	2	− 2.0	2636.0	2	− 3.1
5569	Fe	2925.9	2	− 3.0	2779.0	2	− 9.2	2631.3	2	+ .5
5624	Fe	2931.3	2	− 9.5	2776.1	2	− 7.3	2632.6	2	− 1.6
5658	Fe	2923.7	2	− 2.5	2770.7	2	− 2.5	2630.0	2	+ .5
5709	Fe	2931.6	2	−11.1	2771.3	2	− 3.8	2628.2	2	+ 1.6
5763	Fe	2925.1	2	− 5.6	2767.5	2	− .9	2624.1	2	+ 4.9
5852	Ne				2769.2	4	− 3.9	2629.6	3	− 1.9
5881	Ne				2768.1	4	− 3.2	2629.9	3	− 2.6
5883	Fe	2924.6	2	− 7.1	2767.5	2	− 2.6			
5934	Fe				2767.1	2	− 2.9	2638.8	2	−12.2
5944	Ne				2763.6	4	+ .4	2629.9	3	− 3.4
6003	Fe	2928.5	2	−12.8	2765.8	2	− 2.6			
6029	Fe							2634.1	1	− 8.8
6065	Fe	2918.9	2	− 4.0	2767.0	2	− 4.5	2624.0	2	+ .9
6074	Ne				2764.2	4	− 1.9	2626.1	3	− 1.3
6096	Ne				2763.6	5	− 1.5	2626.1	3	− 1.6
6136	Fe	2915.2	2	− 1.2	2766.0	2	− 4.4	2623.4	2	+ .7
6143	Ne				2764.9	5	− 3.4	2625.9	3	− 1.9
6163	Ne				2762.1	5	− .8	2626.6	3	− 2.8
6191	Fe	2913.3	2	− .2	2767.0	3	− 6.1	2620.7	2	+ 2.7
6217	Ne				2761.5	5	− .9	2625.7	3	− 2.6
6219	Fe				2762.9	1	− 2.4			
6230	Fe				2767.9	1	− 7.5			
6246	Fe				2767.3	1	− 7.1			
6252	Fe	2912.6	2	− .2	2760.7	2	− .5	2620.7	2	+ 2.0
6266	Ne				2764.9	6	− 5.0	2624.8	3	− 2.3
6297	Fe	2904.0	2	+ 7.9	2746.5	2	+13.2			
6304	Ne				2757.7	6	+ 1.9	2623.8	3	− 1.6
6318	Fe				2767.4	1	− 7.9			
6334	Ne				2761.6	7	− 2.3	2624.7	3	− 2.8
6335	Fe	2910.6	2	+ .8	2758.1	2	+ 1.1	2621.0	2	+ .9
6382	Ne				2763.6	7	− 4.9	2624.0	3	− 2.7
6393	Fe	2911.1	2	− .3	2763.7	7	− 5.1	2620.3	4	+ .9
6400	Fe				2765.1	3	− 6.6			
6402	Ne				2763.2	6	− 4.7	2623.9	3	− 2.8
6430	Fe	2910.4	4	− .2	2761.4	7	− 3.2	2615.3	4	+ 5.5
6462	Fe	2905.5	2	+ 4.5	2769.0	1	−11.0	2621.8	2	− 1.2
6494	Fe	2907.0	2	+ 2.6	2761.8	6	− 4.2	2618.0	2	+ 2.2
6506	Ne				2762.2	6	− 4.7	2521.5	3	− 1.4
6532	Ne				2761.0	7	− 3.8	2622.7	3	− 2.8
6546	Fe	2909.0	4	+ .1	2761.0	7	− 4.0	2619.8	4	.0

TABLE 2—Continued

λA	Source	Observations, 0° C, 760 mm	Number of observations	Computed minus observed	Observations, 15° C, 760 mm	Number of observations	Computed minus observed	Observations, 30° C, 760 mm	Number of observations	Computed minus observed
6563	H				2760.2	1	− 3.3			
6598	Ne				2760.2	7	− 3.6	2619.6	3	− 0.3
6609	Fe	2916.9	2	− 8.6	2762.8	1	− 6.4	2612.5	2	+ 6.6
6663	Fe	2913.6	2	− 5.8	2759.6	3	− 3.7	2627.4	2	− 8.8
6678	Ne				2754.6	7	+ 1.2			
6678	Fe	2907.5	2	+ .1	2760.2	6	− 4.4	2618.2	7	+ .3
6717	Ne				2760.2	7	− 4.8	2618.1	3	+ .0
6750	Fe	2901.9	2	− 5.1	2762.4	3	− 7.3	2626.8	2	− 9.0
6752	A				2756.0	2	− .9			
6841	Fe	2907.0	2	− 1.0						
6843	Fe							2609.4	2	+ 7.6
6871	A				2750.9	2	+ 3.1			
6916	Fe							2633.3	2	−16.8
6929	Ne				2757.6	7	− 4.0	2618.4	3	− 2.0
6937	A				2754.5	2	− 1.1			
6945	Fe	2899.7	2	+ 5.4	2749.3	5	+ 4.1	2621.3	2	− 5.2
6965	A				2748.7	2	+ 4.6			
6978	Fe	2903.9	2	+ 1.0	2750.8	3	+ 2.4	2614.5	2	+ 1.5
7016	Fe				2765.4	1	−12.5			
7023	Fe	2909.1	2	− 4.8						
7030	A				2749.0	2	+ 3.7			
7032	Ne				2757.1	8	− 4.4	2617.4	3	− 1.9
7038	Fe							2610.1	2	+ 5.4
7059	Ne				2737.0	1	− 4.5			
7067	A				2748.7	2	+ 3.7			
7068	Fe	2902.4	2	+ 1.6	2767.1	1	−14.7	2624.1	2	− 8.8
7090	Fe	2905.3	2	− 1.5						
7130	Fe	2898.7	2	+ 4.8	2757.6	3	− 5.6	2609.0	2	+ 5.8
7147	A				2749.9	2	+ 2.0			
7164	Fe	2895.0	2	+ 8.3	2761.2	3	− 9.4	2613.6	2	+ 1.0
7173	Ne				2756.7	4	− 5.0			
7187	Fe				2749.3	4	+ 2.3	2626.4	2	−11.9
7207	Fe				2752.9	3	− 1.5			
7223	Fe	2905.7	2	− 3.0				2608.9	2	+ 5.2
7245	Ne				2752.6	6	− 1.5	2616.6	3	− 2.6
7272	A				2748.8	2	+ 2.2			
7293	Fe	2914.5	2	−12.3				2622.4	2	− 8.7
7372	A				2751.2	2	− .9			
7383	A				2746.6	2	+ 3.6			
7389	Fe	2897.7	2	+ 3.8				2611.8	2	+ 1.3
7411	Fe	2900.6	4	+ .7	2751.9	5	− 2.0	2620.2	4	− 7.3
7438	Ne				2742.4	3	+ 7.4			
7445	Fe	2903.6	4	− 2.6	2756.2	3	− 6.5	2609.4	4	+ 3.2
7495	Fe	2906.8	4	− 6.1	2764.5	5	−15.1	2612.7	4	− .3
7503	A				2748.3	2	+ 1.1			
7511	Fe				2749.7	4	− .4			
7514	A				2748.9	2	+ .4			
7531	Fe	2897.2	4	+ 3.3	2740.7	2	+ 8.5	2616.6	4	− 4.4
7568	Fe							2606.8	2	+ 5.1
7586	Fe	2903.0	4	− 3.1				2616.6	2	− 4.7
7620	Fe				2748.1	4	+ .5	2594.7	2	+ 6.9
7635	A				2746.1	2	+ 2.4			
7664	Fe	2895.8	4	+ 3.8	2748.1	3	+ .3	2611.2	4	+ .1
7710	Fe							2611.2	2	− .1
7724	A				2744.9	2	+ 3.1			
7748	Fe	2890.2	4	+ 8.9	2741.7	4	+ 6.2	2603.1	4	+ 7.7
7780	Fe	2897.4	4	+ 1.5	2747.3	3	+ .4	2602.9	4	+ 7.7
7832	Fe	2902.3	4	− 3.8	2750.6	2	− 3.2	2611.2	4	− .8
7937	Fe	2887.8	4	+10.1	2745.5	3	+ 1.4	2615.4	2	− 5.6
7945	Fe	2904.9	2	− 7.0	2747.1	4	− .3	2612.7	2	− 2.9
7948	A				2745.4	2	+ 1.4			
7998	Fe	2886.7	2	+10.4	2748.7	2	− 2.2	2606.6	2	+ 2.9
8006	A				2745.8	2	+ .6			
8014	A				2743.6	2	+ 2.8			
8046	Fe	2890.3	2	+ 7.2	2750.1	2	− 3.8	2605.6	2	+ 3.6

TABLE 2—Continued

λA	Source	Observations, 0° C, 760 mm	Number of observations	Computed minus observed	Observations, 15° C, 760 mm	Number of observations	Computed minus observed	Observations, 30° C, 760 mm	Number of observations	Computed minus observed
8085	Fe	2902.6	2	− 5.5	2739.2	2	+ 6.9	2610.0	2	− 0.9
8103	A				2742.8	1	+ 3.2			
8115	A				2742.0	1	+ 3.9			
8220	Fe	2895.2	2	+ 1.2	2736.1	2	+ 9.3	2612.2	3	− 3.8
8264	A				2742.5	1	+ 2.7			
8327	Fe	2894.3	2	+ 1.5	2738.8	3	+ 6.1	2606.6	3	+ 1.3
8387	Fe	2890.4	2	+ 5.1	2741.2	3	+ 3.4	2599.3	3	+ 8.4
8408	A				2743.6	1	+ .9			
8424	A				2742.7	1	+ 1.7			
8468	Fe	2900.1	2	− 5.0				2606.6	3	+ .7
8514	Fe				2745.9	1	− 1.8	2604.2	3	+ 2.8
8521	A				2746.3	1	− 2.3			
8611	Fe							2599.0	1	+ 7.7
8661	Fe	2881.0	2	+13.2	2740.7	1	+ 2.7	2601.0	4	+ 5.5
8674	Fe							2612.6	1	− 6.1
8688	Fe	2887.3	2	+ 6.7	2734.1	2	+ 9.2	2604.8	4	+ 1.6
8824	Fe				2735.5	1	+ 7.2	2605.4	4	+ 1.2
8999	Fe				2735.6	1	+ 6.5	2591.5	2	+14.8

3. THE CAUCHY DISPERSION FORMULA

The empirical Cauchy formula

$$n-1=a+\frac{b}{\lambda^2}+\frac{c}{\lambda^4}$$

was used to represent these observations of refractivity as a function of the wave length. All the measurements on the refractivity of air at a particular temperature were grouped at intervals of 200 A with the aid of the dispersion formula given by Miss Howell [43]. This gave 34 observational equations from which were obtained the three normal equations required to solve for the three constants of the Cauchy formula by the method of least squares. This solution was made independently for the observations at each of the three temperatures and gave the following equations:

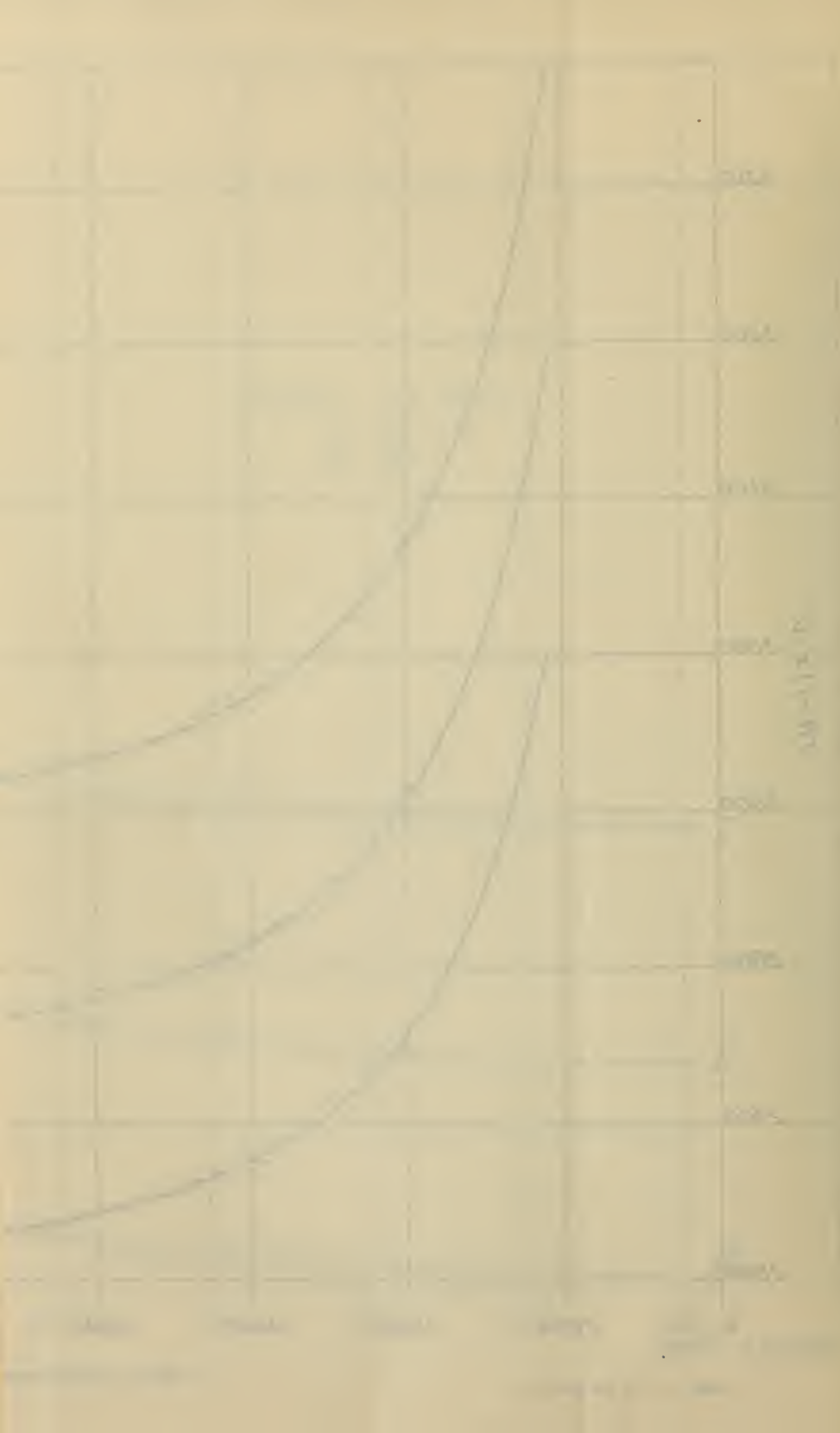
$$0^\circ\text{ C},\ (n-1)\ 10^7=2875.66+\frac{13.412}{\lambda^2\times 10^{-8}}+\frac{0.3777}{\lambda^4\times 10^{-16}}$$

$$15^\circ\text{ C},\ (n-1)\ 10^7=2726.43+\frac{12.288}{\lambda^2\times 10^{-8}}+\frac{0.3555}{\lambda^4\times 10^{-16}}$$

$$30^\circ\ C,\ (n-1)\ 10^7=2589.72+\frac{12.259}{\lambda^2\times 10^{-8}}+\frac{0.2576}{\lambda^4\times 10^{-16}}$$

These equations are represented graphically in Fig. 7.

[43] Howell, Phys. Rev., 6, p. 81; 1915.



The algebraic sum of differences of the grouped observations from the equation values is zero, but this is not quite true of the residuals of the individual observations as given in Table 2. The reason for this is that equal weight was given to the 34 values which represented the observations in 200 A intervals whereas some of them represented slightly more concordant, or a larger number of observations than others.

A total of 1165 index measurements were made:

277 for 122 different wave lengths in air at 0° C,

513 for 185 different wave lengths in air at 15° C,

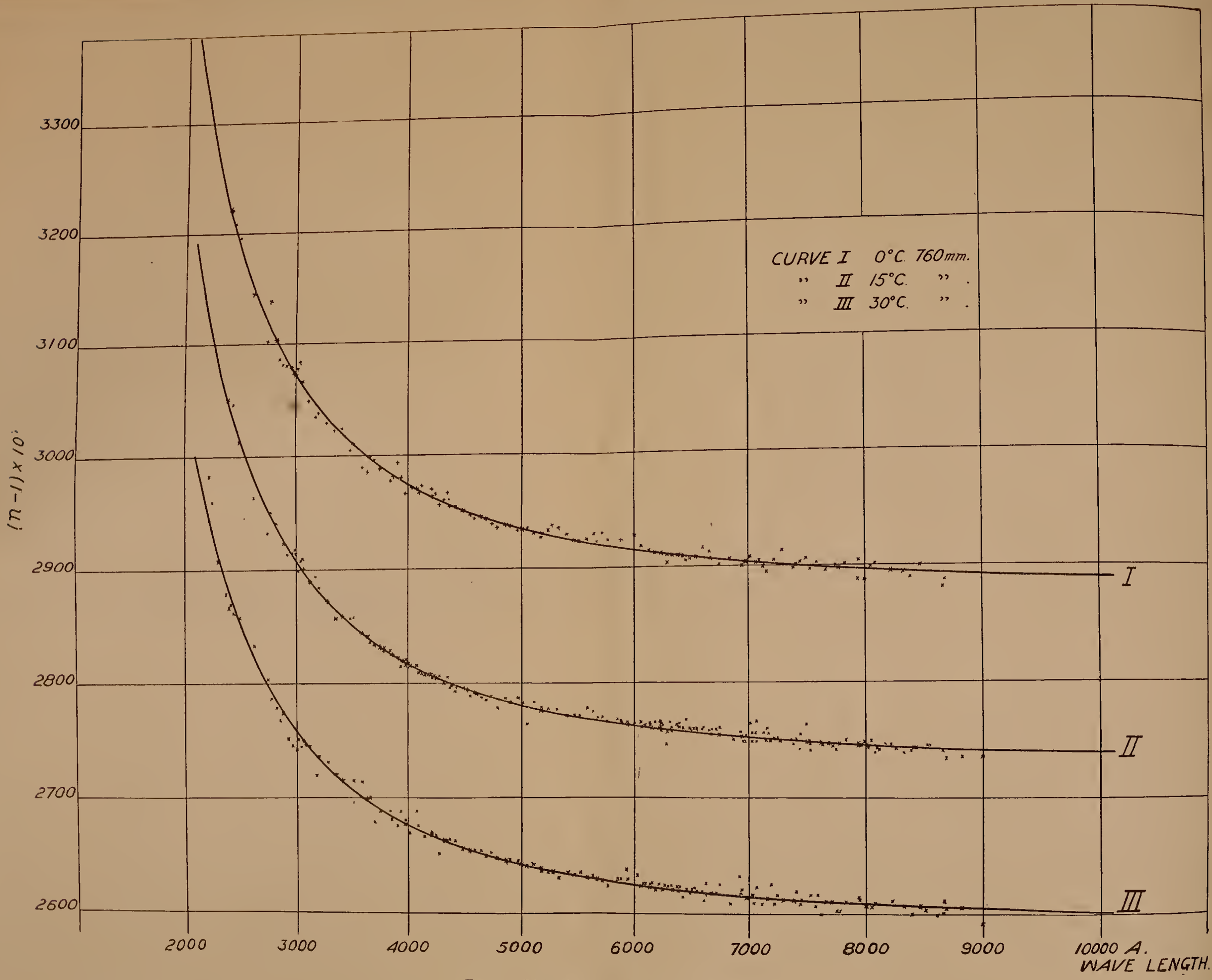
and 375 for 154 different wave lengths in air at 30° C.

In view of the large number of observations which are fairly well distributed throughout the spectrum and quite concordant it is considered that their collection to 34 points, as explained above, gives dispersion equations which are a sufficiently accurate representation of the observations. The deviations of the individual observations from the computed curves indicate a probable error of 3.8 in a single $(n-1)\ 10^7$ for air at 0° C, 3.1 for air at 15° C, and 2.9 for air at 30° C.

4. SOURCES OF ERROR

Although the normal volume percentage of carbon dioxide in the atmosphere at the earth's surface is 0.03, the amount present in even a well-ventilated laboratory with persons present may be twice as large. The index of refraction of carbon dioxide for visible light is about 1.00045 so that the normal amount in the atmosphere contributes little more than one unit in the seventh decimal place of the index of refraction of air. The error introduced in the measurement of the index of refraction of air in a laboratory containing as much as twice the normal amount of carbon dioxide is, therefore, even less than one unit in the seventh decimal place since the increase in carbon dioxide is accompanied by a decrease in oxygen whose index is about 1.00027.

A source of error which has not been mentioned in previous applications of the interferometer to measurement of indices of refraction was called to our attention by Dr. W. F. G. Swann and is due to different dimensions of the apparatus under different pressures. The etalon posts separating the plates of the Fabry and Perot interferometer are larger in a vacuum than they are when surrounded by air at atmospheric pressure. The bulk modulus of invar indicates that the length of these posts is about 2 parts in 3 000 000 less in air than in vacuum. If this compres-



110990°—19. (To face page 723.)

FIG. 7.—*Dispersion curves for dry air Dispersion*

sion is not taken into account in the calculations, the value of the index of refraction will be about one unit too small in the seventh decimal place. However, this correction can not be applied definitely since the effect of similar compression on other parts of the apparatus holding the interferometer plates can not be computed accurately.

It was, at first, thought possible that an error might be introduced by differences of phase change at reflection from the interferometer plates in air and vacuum. If the film of adsorbed gas on the interferometer plates is different at different pressures, it might become evident as a change in the phase correction to wave lengths. Etalons of various sizes from 3 to 25 mm were used in these experiments and the differences in phase change at reflection suffered by different wave lengths were found by a method previously described.[44] The same relative phase changes were found with low pressures as with atmospheric pressures of air on the plates and the absolute phase changes may be assumed to remain constant also. Furthermore, experiments on contact differences of potential show that films of adsorbed gas are quite stable and only show changes after several hours in very high vacuum.

It appears that the systematic errors which affect these measurements of the refractive index of air are extremely small and more or less compensating. The index is increased slightly by the extra carbon dioxide in laboratory air but this increase is probably of the same order of magnitude as the decrease due to the compression of the etalon. It seems safe to assume, therefore, that these systematic errors in our measurements of the index of refraction of air are entirely negligible.

VII. APPLICATIONS

1. CORRECTION OF WAVE LENGTH MEASUREMENTS MADE IN AIR AT OTHER THAN NORMAL TEMPERATURES AND PRESSURES

The international wave length standards are represented by specified radiations whose wave lengths were measured in dry air at 15° C exerting a barometric pressure equivalent to 760 mm of mercury.[45] These standards serve as a basis for the precise measurement of all other wave lengths. Variations in the density of the air appreciably affect the absolute values of the wave lengths and when wave length measurements are made in air whose temperature is not 15° C, or whose pressure is not equal to 760 mm,

[44] This Bulletin, 12, p. 198; 1915–1916.

[45] Astroph. Jl., 32, p. 215, 1910: 33, p. 85, 1911; 39, p. 93, 1914.

corrections must be applied to the measured values to reduce them to their values in air under normal conditions. If the index of refraction of air were a constant throughout the spectrum, all the wave lengths would vary in the same ratio, but the dispersion of the air makes this variation a function of the absolute index and of the density of the air at the time the wave length comparisons were made. For example, let a wave length λ be measured in terms of a standard wave length λ' and consider the result if the measurement is made in air whose density is greater than that defined by the normal conditions. The indices of refraction n and n' for these two wave lengths increase proportionately with the density of the air and the wave lengths decrease. But if λ is smaller than λ', the absolute index of refraction n is larger than n' and the increase in index due to the increased density of air will be a larger amount for n than for n'. Consequently λ is proportionately shorter than λ' in denser air and requires a positive correction to make it comparable with λ' in air at 15° C and 760 mm.

The effect of the temperature and pressure of the air must be taken into account in all accurate comparisons of wave lengths made either by the coincidence method with diffraction gratings[46] or with interferometers.[47] The necessary corrections are generally small and may be negligible under certain conditions. In order actually to calculate these corrections let us imagine the wavelength comparisons to be made by means of an interferometer in air whose density, d, is defined by the temperature t and the barometric pressure B. Let λ represent the wave length of a radiation under these conditions, n the index of refraction of this air for this wave length, and λ' and n' analogous quantities for the standard wave length with which λ is to be compared. The same letters with the subscript o may be used to represent the same quantities under standard observing conditions; that is, air at 15° C and 760 mm. It is desired to obtain λ_o. The orders of interference measured under the actual conditions give $p\,\lambda = p'\,\lambda'$ $(=2e)$. The calculation for λ, however, is made from $\frac{p'\,\lambda'_o}{p}$ in which the standard wave length is used as if the air had no dispersion, and then a correction is applied.

[46] Kayser, Handbuch der Spectroscopie, 1, p. 719; 1900.
[47] Buisson and Fabry, Jour. de Phys., 7, p. 169; 1908.

This correction, the difference between the exact value and that calculated, is equal to

$$\delta = \lambda_o - \frac{p'\lambda'_o}{p} = \lambda_o\left(1 - \frac{p'\lambda'_o}{p\lambda_o}\right) = \lambda_o\left(1 - \frac{\lambda\lambda'_o}{\lambda'\lambda_o}\right)$$

But

$$\frac{\lambda}{\lambda_o} = \frac{n_o}{n} \text{ and } \frac{\lambda'}{\lambda'_o} = \frac{n_o'}{n'}$$

Then

$$\delta = \lambda_o\left(1 - \frac{n_o n'}{n_o' n}\right) = \lambda_o \frac{n_o' n - n_o n'}{n_o' n} = \lambda_o(n_o' n - n_o n')$$

since $n_o' n$ in the denominator may be called unity. Assuming the refractivity to be proportional to the density

$$\frac{n-1}{d} = \frac{n_o - 1}{d_o} \text{ and } \frac{n'-1}{d} = \frac{n_o' - 1}{d_o}$$

we have

$$n = \frac{(n_o - 1)d}{d_o} + 1 \text{ and } n' = \frac{(n_o' - 1)d}{d_o} + 1$$

and the correction reduces to

$$\delta = \lambda_o(n_o - n_o') \frac{d - d_o}{d_o}$$

The factor $\frac{d-d_o}{d_o}$ is easily calculated as a function of temperature and pressure, and is constant for all the waves compared under these conditions.

The quantity $\lambda_o\ (n_o - n_o')$ may be calculated as a function of the wave length and represented by a curve from which the correction to the wave length can be obtained by multiplying its ordinate by the appropriate density factor $\frac{d-d_o}{d_o}$.

In order to avoid all these troublesome computations hereafter, Tables 3 and 4 have been prepared to give these corrections for the entire range of wave lengths and densities of air in which accurate spectroscopic measurements are ordinarily made. Table 3 gives the density factor $\frac{d-d_o}{d_o}$ in terms of the temperature t of the

air in centigrade degrees and the barometric pressure B of the air in millimeters of mercury.

Table 4 gives the corrections, $\delta = \lambda_o (n_o - n_o') \frac{a - d_o}{d_o}$, in ten-thousandths of an Angstrom. These tables are constructed for the correction of wave length measurements in terms of the fundamental spectroscopic standard; that is, the wave length 6438.4696 A of cadmium red radiation in dry air at 15° C and 760 mm. The density factor (Table 3) is therefore zero for $t = 15°$ C and $B = 760$ mm., and the correction (Table 4) is always zero for $\lambda = 6438$ A.

To illustrate the use of these tables, find the correction required by λ when it is measured as 3000.0000 A in air at 25° C and 720 mm. Table 3 gives $\frac{d - d_o}{d_o} = -0.085$ and this in Table 4 gives a correction of −0.0038 A to λ. Again, if λ, under the same atmospheric conditions, is measured as 8000.0000 A in terms of a standard λ' of wave length 4000.0000 A, say, the measurement will require a correction of (0.0020 + 0.0008 =) +0.0028 A. By easy interpolation in these tables the corrections can be found for all wave lengths between 2000 A and 10 000 A when the wave length comparisons are made in air whose density is determined by a temperature range from 9 to 35° C and a pressure range from 600 to 780 mm. of mercury.

2. CORRECTIONS FOR WATER VAPOR IN THE AIR

The refractive index of water vapor has not been fully investigated and the corrections necessary to change wave lengths measured in wet air to their values in dry air are therefore not very accurately known. Lorenz measured the index of refraction of water vapor (specific gravity = 0.0008061) for sodium light as 1.0002500. From this result he recommends a correction of $+0.000041 \frac{m}{760}$ to indices of refraction measured in air containing water vapor of m millimeters pressure.

No measurements on the dispersion of water vapor are known to exist, but if the dispersion is assumed to be comparable with that of air, the corrections to relative wave lengths will not be affected, since they depend on the difference of indices $n_o - n_o'$.

Two of our experiments on the dispersion of air for waves between 5800 A and 7500 A were made with air having a water vapor pressure of 13 mm. The index of refraction was observed to be diminished by approximately the amount given by the Lorenz

TABLE 3.—$\frac{d-d_0}{d_0}$

B mm	t 9° C	t 11° C	t 13° C	t 15° C	t 17° C	t 19° C	t 21° C	t 23° C	t 25° C	t 27° C	t 29° C	t 31° C	t 33° C	t 35° C
600	−0.192	−0.200	−0.206	−0.211	−0.216	−0.222	−0.227	−0.232	−0.238	−0.243	−0.248	−0.253	−0.258	−0.262
605	− .186	− .193	− .199	− .205	− .210	− .215	− .221	− .225	− .232	− .236	− .242	− .246	− .252	− .256
610	− .179	− .187	− .192	− .198	− .204	− .208	− .214	− .219	− .225	− .230	− .235	− .240	− .246	− .250
615	− .172	− .180	− .186	− .191	− .197	− .202	− .208	− .212	− .219	− .224	− .229	− .234	− .239	− .244
620	− .166	− .173	− .179	− .185	− .190	− .196	− .202	− .206	− .212	− .217	− .223	− .228	− .233	− .237
625	− .160	− .167	− .172	− .178	− .184	− .189	− .195	− .200	− .206	− .211	− .216	− .222	− .227	− .231
630	− .153	− .160	− .166	− .171	− .177	− .182	− .188	− .194	− .200	− .205	− .210	− .215	− .221	− .225
635	− .146	− .153	− .159	− .165	− .171	− .176	− .182	− .187	− .193	− .199	− .204	− .209	− .214	− .219
640	− .140	− .147	− .152	− .158	− .164	− .170	− .176	− .180	− .187	− .192	− .198	− .203	− .208	− .213
645	− .133	− .140	− .146	− .151	− .158	− .163	− .169	− .174	− .181	− .186	− .191	− .197	− .202	− .206
650	− .126	− .133	− .139	− .145	− .151	− .156	− .163	− .168	− .174	− .179	− .185	− .190	− .196	− .200
655	− .119	− .127	− .132	− .138	− .145	− .150	− .156	− .162	− .168	− .173	− .179	− .184	− .190	− .194
660	− .112	− .120	− .126	− .131	− .138	− .144	− .150	− .155	− .162	− .166	− .172	− .178	− .184	− .188
665	− .105	− .113	− .119	− .125	− .132	− .137	− .144	− .149	− .155	− .160	− .166	− .172	− .177	− .182
670	− .099	− .107	− .113	− .119	− .125	− .131	− .137	− .143	− .149	− .154	− .160	− .165	− .171	− .176
675	− .092	− .100	− .106	− .112	− .118	− .124	− .130	− .136	− .143	− .148	− .154	− .159	− .165	− .170
680	− .086	− .093	− .099	− .105	− .112	− .118	− .124	− .129	− .136	− .142	− .147	− .153	− .159	− .163
685	− .079	− .087	− .093	− .098	− .106	− .111	− .117	− .123	− .130	− .135	− .141	− .147	− .153	− .157
690	− .072	− .080	− .086	− .092	− .099	− .105	− .111	− .116	− .123	− .129	− .135	− .140	− .146	− .151
695	− .065	− .073	− .079	− .085	− .092	− .098	− .105	− .110	− .117	− .122	− .128	− .134	− .140	− .145
700	− .059	− .067	− .073	− .079	− .086	− .092	− .098	− .104	− .111	− .116	− .122	− .128	− .134	− .139
705	− .052	− .060	− .066	− .072	− .080	− .086	− .092	− .097	− .104	− .110	− .115	− .122	− .127	− .133
710	− .046	− .053	− .060	− .066	− .073	− .079	− .085	− .091	− .098	− .104	− .109	− .116	− .121	− .127
715	− .039	− .047	− .053	− .059	− .067	− .073	− .079	− .084	− .091	− .097	− .103	− .110	− .115	− .121
720	− .032	− .040	− .046	− .053	− .060	− .066	− .072	− .078	− .085	− .091	− .097	− .103	− .109	− .114
725	− .026	− .033	− .040	− .046	− .054	− .060	− .066	− .072	− .079	− .085	− .090	− .097	− .103	− .108
730	− .019	− .027	− .033	− .039	− .047	− .053	− .059	− .065	− .072	− .078	− .084	− .091	− .097	− .102
735	− .012	− .020	− .026	− .033	− .040	− .047	− .053	− .059	− .066	− .072	− .078	− .085	− .090	− .096
740	− .005	− .013	− .020	− .026	− .034	− .040	− .046	− .052	− .060	− .066	− .072	− .078	− .084	− .090
745	+ .001	− .007	− .013	− .020	− .027	− .034	− .040	− .046	− .053	− .059	− .065	− .072	− .078	− .084
750	+ .008	± .000	− .007	− .013	− .021	− .027	− .033	− .040	− .047	− .053	− .059	− .066	− .072	− .077
755	+ .015	+ .007	± .000	− .007	− .014	− .021	− .027	− .033	− .041	− .047	− .053	− .060	− .064	− .071
760	+ .022	+ .013	+ .006	± .000	− .008	− .014	− .021	− .027	− .034	− .040	− .046	− .054	− .059	− .065
765	+ .028	+ .020	+ .013	+ .006	− .001	− .008	− .014	− .020	− .028	− .034	− .040	− .047	− .053	− .059
770	+ .035	+ .027	+ .020	+ .013	+ .005	− .001	− .008	− .014	− .022	− .028	− .034	− .041	− .047	− .053
775	+ .042	+ .033	+ .026	+ .020	+ .012	+ .005	− .001	− .008	− .015	− .021	− .028	− .035	− .041	− .047
780	+ .048	+ .040	+ .033	+ .026	+ .019	+ .012	+ .005	− .001	− .009	− .015	− .021	− .029	− .034	− .041

TABLE 4.—$\delta=\lambda_o(n_o-n_o')\frac{d-d_o}{d_o}$(in ten-thousandths A)

$\frac{d-d_o}{d_o}$	λ 2000	λ 2500	λ 3000	λ 3500	λ 4000	λ 4500	λ 5000	λ 5500	λ 6000	λ 6500	λ 7000	λ 7500	λ 8000	λ 8500	λ 9000	λ 9500	λ 10 000
−0.260	−259	−166	−116	−84	−61	−44	−30	−18	−8	+1	+9	+17	+24	+31	+37	+44	+50
−0.250	−249	−160	−112	−81	−59	−42	−29	−18	−8	+1	+9	+16	+23	+30	+36	+42	+48
−0.240	−239	−154	−107	−78	−57	−41	−28	−17	−7	+1	+9	+16	+22	+29	+35	+40	+46
−0.230	−229	−147	−103	−74	−54	−39	−27	−16	−7	+1	+8	+15	+21	+27	+33	+39	+44
−0.220	−219	−141	− 98	−71	−52	−37	−26	−15	−7	+1	+8	+14	+20	+26	+32	+37	+42
−0.210	−209	−134	− 94	−68	−50	−35	−24	−15	−7	+1	+8	+14	+20	+25	+30	+35	+40
−0.200	−199	−128	− 89	−65	−47	−34	−23	−14	−6	+1	+7	+13	+19	+24	+29	+34	+38
−0.190	−189	−122	− 85	−61	−45	−32	−22	−13	−6	+1	+7	+12	+18	+23	+27	+32	+36
−0.180	−179	−115	− 80	−58	−42	−30	−21	−13	−6	+1	+6	+12	+17	+21	+26	+30	+34
−0.170	−169	−109	− 76	−55	−40	−29	−20	−12	−5	+1	+6	+11	+16	+20	+24	+29	+32
−0.160	−159	−102	− 71	−52	−38	−27	−19	−11	−5	+1	+6	+10	+15	+19	+23	+27	+31
−0.150	−149	− 96	− 67	−48	−35	−25	−17	−11	−5	+1	+5	+10	+14	+18	+22	+25	+29
−0.140	−139	− 90	− 62	−45	−33	−24	−16	−10	−4	+1	+5	+ 9	+13	+17	+20	+24	+27
−0.130	−129	− 83	− 58	−42	−31	−22	−15	− 9	−4	+1	+5	+ 9	+12	+15	+19	+22	+25
−0.120	−119	− 77	− 54	−39	−28	−20	−14	− 8	−4	+0	+4	+ 8	+11	+14	+17	+20	+23
−0.110	−109	− 70	− 49	−36	−26	−19	−13	− 8	−3	+0	+4	+ 7	+10	+13	+16	+18	+21
−0.100	−100	− 64	− 45	−32	−24	−17	−12	− 7	−3	+0	+4	+ 7	+ 9	+12	+14	+17	+19
−0.090	− 90	− 58	− 40	−29	−21	−15	−10	− 6	−3	+0	+3	+ 6	+ 8	+11	+13	+15	+17
−0.080	− 80	− 51	− 36	−26	−19	−14	− 9	− 6	−2	+0	+3	+ 5	+ 7	+10	+12	+13	+15
−0.070	− 70	− 45	− 31	−23	−17	−12	− 8	− 5	−2	+0	+3	+ 5	+ 7	+ 8	+10	+12	+13
−0.060	− 60	− 38	− 27	−19	−14	−10	− 7	− 4	−2	+0	+2	+ 4	+ 6	+ 7	+ 9	+10	+11
−0.050	− 50	− 32	− 22	−16	−12	− 8	− 6	− 4	−2	+0	+2	+ 3	+ 5	+ 6	+ 7	+ 8	+10
−0.040	− 40	− 26	− 18	−13	− 9	− 7	− 5	− 3	−1	+0	+1	+ 3	+ 4	+ 5	+ 6	+ 7	+ 8
−0.030	− 30	− 19	− 13	−10	− 7	− 5	− 3	− 2	−1	+0	+1	+ 2	+ 3	+ 4	+ 4	+ 5	+ 6
−0.020	− 20	− 13	− 9	− 6	− 5	− 3	− 2	− 1	−1	+0	+1	+ 1	+ 2	+ 2	+ 3	+ 3	+ 4
−0.010	− 10	− 6	− 4	− 3	− 2	− 2	− 1	− 1	−0	+0	+0	+ 1	+ 1	+ 1	+ 1	+ 2	+ 2
±0.000	± 0	± 0	± 0	± 0	± 0	± 0	± 0	± 0	±0	±0	±0	± 0	± 0	± 0	± 0	± 0	± 0
+0.010	+ 10	+ 6	+ 4	+ 3	+ 2	+ 2	+ 1	+ 1	+0	−0	−0	− 1	− 1	− 1	− 1	− 2	− 2
+0.020	+ 20	+ 13	+ 9	+ 6	+ 5	+ 3	+ 2	+ 1	+1	−0	−1	− 1	− 2	− 2	− 3	− 3	− 4
+0.030	+ 30	+ 19	+ 13	+10	+ 7	+ 5	+ 3	+ 2	+1	−0	−1	− 2	− 3	− 4	− 4	− 5	− 6
+0.040	+ 40	+ 26	+ 18	+13	+ 9	+ 7	+ 5	+ 3	+1	−0	−1	− 3	− 4	− 5	− 6	− 7	− 8
+0.050	+ 50	+ 32	+ 22	+16	+12	+ 8	+ 6	+ 4	+2	−0	−2	− 3	− 5	− 6	− 7	− 8	−10

expression. Furthermore, this decrease was about the same for different wave lengths. These results on wet air are not complete enough to add much to the subject of dispersion of light by water vapor, but they show that the corrections to index of refraction as recommended above for water vapor in the air are quite proper.

3. CORRECTIONS TO CONVERT TO VACUUM VALUES

Corrections required to convert wave lengths or oscillation frequencies measured in air at 20° C and 760 mm to their value in vacuum are found in Watts' Index of Spectra, appendix E, page 51, and in Kayser's Handbuch der Spectroscopie, II, page 513. These are applicable to Rowland's standard wave lengths and all wave lengths determined from them, but must be revised to apply to the international system of wave lengths which represent values measured in air at 15° C and 760 mm.

The dispersion equation representing our observations on the indices of refraction of air at 15° C and 760 mm was used to construct a new table of corrections. Table 5 gives the values of wave length in air, (λ), refractivity, $(n-1)10^7$, vacuum correction to λ, ($n\lambda-\lambda$), oscillation frequency in air, $\left(\frac{1}{\lambda}\right)$, and vacuum correction to frequency, $\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$, from 2000 A to 10 000 A at intervals of 50 A. This table shows, for example, that a light wave of length 5000.000 A in dry air at 15° C and 760 mm is increased in length by 1.391 A in a vacuum. The frequency (number of waves per centimeter) in air is 20 000.00, but in a vacuum it is diminished by 5.56. The vacuum frequencies corresponding to any wave lengths in the international scale can be readily determined by the use of Table 5 together with a table of reciprocals with seven place arguments.

4. ASTRONOMICAL REFRACTION

Newcomb states in his Compendium of Spherical Astronomy, page 223:

> There is, perhaps, no branch of practical astronomy on which so much has been written as on this (astronomical refraction) and which is still in so unsatisfactory a state. The difficulties connected with it are both theoretical and practical. The theoretical difficulties arise from the uncertainty and variability of the law of diminution of the density of the atmosphere with height, and also from the mathematical difficulty of integrating the equations of the refraction for altitudes near the horizon, after the best law of diminution has been adopted.

TABLE 5.—Table of corrections to be Applied to Wave Lengths in Air at 15° C and 760 mm to Reduce Wave Lengths and Frequencies to Vacuum Values

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ subtract
2000	3255.82	0.6512	50 000	16.274
2050	3220.12	.6601	48 780	15.703
2100	3187.86	.6695	47 619	15.175
2150	3158.63	.6791	46 511	14.687
2200	3132.07	.6891	45 454	14.232
2250	3107.87	.6993	44 444	13.808
2300	3085.75	.7097	43 478	13.412
2350	3065.50	.7204	42 553	13.041
2400	3046.91	.7313	41 666	12.692
2450	3029.81	.7423	40 816	12.363
2500	3014.05	.7535	40 000	12.053
2550	2999.48	.7649	39 215	11.759
2600	2986.00	.7764	38 461	11.481
2650	2973.50	.7880	37 735	11.217
2700	2961.88	.7997	37 037	10.967
2750	2951.08	.8115	36 363	10.728
2800	2941.00	.8235	35 714	10.500
2850	2931.60	.8355	35 087	10.283
2900	2922.80	.8476	34 482	10.076
2950	2914.57	.8598	33 898	9.877
3000	2906.85	.8721	33 333	9.687
3050	2899.60	.8844	32 786	9.504
3100	2892.79	.8968	32 258	9.329
3150	2886.38	.9092	31 746	9.160
3200	2880.33	.9217	31 250	8.998
3250	2874.63	.9343	30 769	8.842
3300	2869.24	.9469	30 303	8.692
3350	2864.15	.9595	29 850	8.547
3400	2859.33	.9722	29 411	8.407
3450	2854.76	.9849	28 985	8.272
3500	2850.43	.9977	28 571	8.142
3550	2846.32	1.0104	28 169	8.016
3600	2842.41	1.0233	27 777	7.893
3650	2838.69	1.0361	27 397	7.774
3700	3835.16	1.0490	27 027	7.660
3750	2831.79	1.0619	26 666	7.550
3800	2828.58	1.0749	26 315	7.442
3850	2825.51	1.0878	25 974	7.337
3900	2822.59	1.1008	25 641	7.235
3950	2819.79	1.1138	25 316	7.137
4000	2817.12	1.1268	25 000	7.041
4050	2814.56	1.1399	24 691	6.948
4100	2812.11	1.1530	24 390	6.857
4150	2809.76	1.1661	24 096	6.769
4200	2807.51	1.1792	23 809	6.683
4250	2805.36	1.1923	23 529	6.599
4300	2803.29	1.2054	23 255	6.517
4350	2801.30	1.2186	22 988	6.437
4400	2799.39	1.2317	22 727	6.360
4450	2797.55	1.2449	22 471	6.285
4500	2795.78	1.2581	22 222	6.211
4550	2794.08	1.2713	21 978	6.139
4600	2792.44	1.2845	21 739	6.069
4650	2790.86	1.2978	21 505	6.000
4700	2789.34	1.3110	21 276	5.933
4750	2787.88	1.3242	21 052	5.868
4800	2786.46	1.3375	20 833	5.804
4850	2785.09	1.3508	20 618	5.741
4900	2783.78	1.3640	20 406	5.680
4950	2782.50	1.3773	20 202	5.620
5000	2781.27	1.3906	20 000	5.561
5050	2780.08	1.4039	19 801	5.503
5100	2778.93	1.4173	19 607	5.447
5150	2777.81	1.4306	19 417	5.392
5200	2776.74	1.4439	19 230	5.336

TABLE 5—Continued

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ subtract
5250	2775.69	1.4572	19 047	5.285
5300	2774.68	1.4706	18 867	5.234
5350	2773.70	1.4839	18 691	5.183
5400	2772.75	1.4973	18 518	5.133
5450	2771.83	1.5106	18 348	5.084
5500	2770.94	1.5240	18 181	5.037
5550	2770.07	1.5374	18 018	4.990
5600	2769.23	1.5508	17 857	4.944
5650	2768.41	1.5642	17 699	4.899
5700	2767.62	1.5775	17 543	4.854
5750	2766.85	1.5909	17 391	4.811
5800	2766.10	1.6043	17 241	4.768
5850	2765.37	1.6177	17 094	4.726
5900	2764.66	1.6311	16 949	4.685
5950	2763.98	1.6446	16 806	4.644
6000	2763.31	1.6580	16 666	4.604
6050	2762.65	1.6714	16 528	4.565
6100	2762.02	1.6848	16 393	4.527
6150	2761.40	1.6983	16 260	4.489
6200	2760.80	1.7117	16 129	4.452
6250	2760.22	1.7251	16 000	4.415
6300	2759.65	1.7386	15 873	4.379
6350	2759.09	1.7520	15 748	4.344
6400	2758.55	1.7655	15 625	4.309
6450	2758.02	1.7789	15 503	4.275
6500	2757.51	1.7924	15 384	4.241
6550	2757.00	1.8058	15 267	4.208
6600	2756.51	1.8193	15 151	4.175
6650	2756.04	1.8328	15 037	4.143
6700	2755.57	1.8462	14 925	4.112
6750	2755.11	1.8597	14 814	4.081
6800	2754.67	1.8732	14 705	4.050
6850	2754.23	1.8866	14 598	4.020
6900	2753.81	1.9001	14 492	3.990
6950	2753.39	1.9136	14 388	3.961
7000	2752.99	1.9271	14 285	3.932
7050	2752.59	1.9406	14 184	3.903
7100	2752.21	1.9541	14 084	3.875
7150	2751.83	1.9676	13 986	3.847
7200	2751.46	1.9811	13 888	3.820
7250	2751.10	1.9945	13 793	3.793
7300	2750.74	2.0080	13 698	3.767
7350	2750.39	2.0215	13 605	3.741
7400	2750.06	2.0350	13 513	3.715
7450	2749.72	2.0485	13 422	3.690
7500	2749.40	2.0620	13 333	3.665
7550	2749.08	2.0756	13 245	3.640
7600	2748.77	2.0891	13 157	3.616
7650	2748.46	2.1026	13 071	3.592
7700	2748.17	2.1161	12 987	3.568
7750	2747.88	2.1296	12 903	3.545
7800	2747.59	2.1431	12 820	3.522
7850	2747.31	2.1566	12 738	3.499
7900	2747.03	2.1702	12 658	3.476
7950	2746.76	2.1837	12 578	3.454
8000	2746.50	2.1972	12 500	3.432
8050	2746.24	2.2107	12 422	3.410
8100	2745.99	2.2243	12 345	3.389
8150	2745.74	2.2378	12 269	3.368
8200	2745.49	2.2513	12 195	3.347
8250	2745.25	2.2648	12 121	3.326
8300	2745.02	2.2784	12 048	3.306
8350	2744.78	2.2919	11 976	3.286
8400	2744.56	2.3054	11 904	3.266
8450	2744.34	2.3190	11 834	3.246

TABLE 5—Continued

λ	$(n-1)10^7$	$(n\lambda-\lambda)$ add	$\frac{1}{\lambda}$	$\left(\frac{1}{n\lambda}-\frac{1}{\lambda}\right)$ subtract
8500	2744. 12	2. 3325	11 764	3. 227
8550	2743. 90	2. 3460	11 695	3. 208
8600	2743. 69	2. 3596	11 627	3. 189
8650	2743. 49	2. 3731	11 560	3. 170
8700	2743. 28	2. 3867	11 494	3. 152
8750	2743. 09	2. 4002	11 428	3. 134
8800	2742. 89	2. 4137	11 363	3. 116
8850	2742. 70	2. 4273	11 299	3. 099
8900	2742. 51	2. 4408	11 235	3. 080
8950	2742. 32	2. 4544	11 173	3. 063
9000	2742. 14	2. 4679	11 111	3. 046
9050	2741. 96	2. 4815	11 049	3. 029
9100	2741. 79	2. 4950	10 989	3. 012
9150	2741. 61	2. 5086	10 928	2. 995
9200	2741. 44	2. 5221	10 869	2. 979
9250	2741. 28	2. 5357	10 810	2. 963
9300	2741. 11	2. 5492	10 752	2. 947
9350	2740. 95	2. 5628	10 695	2. 931
9400	2740. 79	2. 5763	10 638	2. 915
9450	2740. 64	2. 5899	10 582	2. 899
9500	2740. 48	2. 6035	10 526	2. 884
9550	2740. 33	2. 6170	10 471	2. 869
9600	2740. 18	2. 6306	10 416	2. 854
9650	2740. 04	2. 6441	10 362	2. 839
9700	2739. 89	2. 6577	10 309	2. 824
9750	2739. 75	2. 6713	10 256	2. 809
9800	2739. 61	2. 6848	10 204	2. 795
9850	2739. 47	2. 6984	10 152	2. 780
9900	2739. 34	2. 7119	10 101	2. 766
9950	2739. 20	2. 7255	10 050	2. 752
10 000	2739. 07	2. 7391	10 000	2. 738

The practical difficulties involve the relation of refractive index with density of air as depending upon pressure, temperature, and humidity, and on the necessary inattention to the dispersion of light. In practice the astronomer usually disregards all dependence of refractive index on color. This must involve errors in zenith distance observations on stars whose colors are intrinsically different and especially in observations of stars near the horizon where the short waves are largely absorbed and scattered by our atmosphere causing the stars to be represented only by longer waves. Because of these difficulties, tables of corrections for astronomical refraction are probably better determined from astronomical observations than from laboratory measures. The refraction tables which have been applied to the major portion of astronomical observations are those of Bessel founded upon the observations of Bradley and published in Tabulae Regiomontanae. In 1870 the Pulkowa tables were published under the title Tabulae Refractionium in usum speculae Pulcovensis Congestae. These are based upon the researches of Gylden[48] and are supplemented

[48] Gylden, Mem. de l'Acad. de St. Petersburg, **10**, No. 1, 1866, and **12**, No. 4, 1868.

by those of Fuss.[49] The Pulkowa tables are founded upon a smaller refraction constant than Bessel's, and Newcomb (loc. cit.) says the most recent discussions and comparisons indicate a still greater diminution in the constant. The constants under discussion represent the white light index of refraction of air at 0° C, 760 mm, as follows:[50]

Bessel	1.00029257, from Bradley.
Gylden	29232, from Pulkowa, 1842–1849
Fuss	29161, from Pulkowa, 1867–1869
Newcomb	29195, from Greenwich, 1877–1886
Bauschinger	29152, from Munchen, 1891–1893
Courvoisier	29180, from Heidleberg, 1900–1903

Under the same conditions our observations give about 1.0002923 assuming the wave length 5560 A of maximum visibility for white light.

For photographic work a larger refraction constant must be used than for visual. If the photographic maximum is at 4200 A the refraction constant should be 1.0002968 and the photographic corrections are therefore 1.0154 larger than the visual. The ratio of these corrections has been found experimentally by Wilsing[51] to be 1.01539.

5. OPTICAL TEMPERATURE COEFFICIENT

If the Dale and Gladstone law, $\frac{n-1}{d}=\text{constant}$, holds for air, the temperature coefficient of index variation should be identical with the temperature coefficient of volume change at constant pressure and any refractivity of air at temperature t° C could be expressed in terms of the refractivity of air at 0° C by the relation

$$(n-1)_t=\frac{(n-1)_o}{1+\alpha t}$$

in which $\alpha=0.00367$.

The following values of the optical temperature coefficient of air show considerable disagreement: Lorenz, 0.00367; Von Lang, 0.00310; Mascart, 0.00383; Benoit, 0.00367; and Walker, 0.00360.

Our observations on the refractivity of air at temperatures of 0, 15, and 30° C show that α is a function of the wave length and increases rapidly as the absorption band in the ultra-violet is approached. This is shown in Table 6.

[49] Fuss, ibid., 18, No. 4; 1868.
[50] Winklemann, Handbuch der Physik, 6 (2). p. 534.
[51] Wilsing, Astronom. Nachr., 145, p. 293; 1898.

TABLE 6

λ	α
8500	0.003672
7500	3674
6500	3678
5500	3685
4500	3700
3500	3738
2500	3872

A decrease in the density of air due to an increase in its temperature either apparently or actually changes the position of the ultra-violet absorption band. As the temperature of the air rises the absorption and dispersion of short waves of light diminish and this effect is probably explained by the increased kinetic energy of the gas molecules rather than by the decreased density of the gas. It would be interesting to test this by ultra-violet dispersion measurements of air with density reductions at constant temperature.

Within the range of our observations the refractivity of air at any temperature for any particular wave length λ can be obtained from measurements on air at 0° C by the relation mentioned above if a correction be applied to α which is a function of λ. A very simple correction to α is contained in the expression

$$(n-1)_t = \frac{(n-1)_0}{1+t\,(\alpha+\frac{3\times 10^6}{\lambda^3})}$$

in which $\alpha = 0.00367$; which allows the calculation of $(n-1)_t$ within the limits of probable error of observation.

6. DISPERSION FORMULAS

A large amount of theoretical work on refraction and dispersion of transparent materials has been done in the past, and various theories have been tested by measurements of indices of refraction of solids and liquids over a wide range of spectrum from the ultra-violet through the visible and far out into the infra-red. It is especially important for this purpose to measure indices of refraction in the neighborhood of absorption bands. Measurements of refractivities of gases have heretofore been confined to a comparatively small portion of the spectrum, and although more work has been done with air than with other gases the review of previous

work showed how incomplete it was. It was hoped that the present investigation of the dispersion of air for red and infra-red light would give an indication of an absorption band in the infra-red so that the Sellmeier or Ketteler-Helmholtz dispersion formula could be tested in its representation of the observations. The empirical Cauchy equation with three constants was first used to represent the measurements, and although the representation was quite satisfactory, there seemed to be a slight deviation in the red which might be explained by an absorption band in the infra-red. An effort was made to locate such a band from our dispersion measurements, but without success.

Water vapor has a very complicated absorption spectrum in the infra-red, but dry air is known to be quite transparent to long waves. The only constituent of air which seems to absorb much infra-red light is carbon dioxide. Although carbon dioxide constitutes only 0.03 per cent of the volume of ordinary air, the absorption is quite marked for wave lengths 42 700 A[52,53] and 147 000 A[54] when these waves are received through several meters of air. Rubens and Wartenburg [55] have found dry air to be quite transparent for wave lengths 230 000 A, 520 000 A, 1 100 000 A, and 3 140 000 A. The absorption bands due to the carbon dioxide in the air are narrow and weak compared to the absorption band of air for ultra-violet light and it seems doubtful that they can exert any marked influence on dispersion. The effect of these bands is probably similar to that of the *A* band due to atmospheric oxygen which strongly absorbs light of wave length 7600 A, but relatively small thicknesses of air freely transmit these waves.

Koch [56] measured the index of refraction of air as 1.00028806 for residual rays from gypsum ($\lambda = 67\ 094$ A) and 1.00028875 for residual rays from calcite ($\lambda = 86\ 784$ A). If no absorption band exists between 9000 A and 67 000 A, and if it is permissible to extrapolate the formula representing our observations, we obtain $n = 1.0002876$ for the longest waves.

It is probably necessary, therefore, to conclude that if there is any marked absorption of infra-red waves by air, the band of absorption must lie at extremely long waves.

Moreover, the electromagnetic theory has connected the index of refraction n of a medium to its dielectric constant D by the

[52] Paschen, Wied. Ann., 53, p. 334; 1894.
[53] Statescu, Phil. Mag., 30, p. 737; 1915.
[54] Rubens, Wied. Ann., 64, p. 584; 1898.
[55] Rubens and Wartenburg, Phys. Zeitschr., 12, p. 1080; 1911.
[56] Koch, Nova Acta Soc. Upsal (4), 2; 1909.

relation $n^2 = D$. This relation was derived for long waves and is not generally satisfied in the visible spectrum where all substances have more or less dispersion. For certain gases, such as hydrogen, nitrogen, and dry air, which show only small dispersion and no strong absorption in the infra-red, $n^2 = D$ approximately, even when n is taken for waves in the visible spectrum.[57] For example,[58] D for dry air is 1.000590 and n^2 for sodium light is 1.000584. The dispersion for such substances is well represented by the simple empirical formula of Cauchy

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$

If n^2 is not equal to D for visible rays, the dispersion can not be represented by Cauchy's formula, but the Sellmeier or Ketteler-Helmholtz formula which takes into account the free periods corresponding to absorption bands of wave length λ_κ must be used. Neglecting absorption for wave lengths λ this formula is generally written as follows:

$$n^2 = 1 + \Sigma \frac{m_\kappa \lambda^2}{\lambda^2 - \lambda_\kappa^2}$$

The effect on n of each species of ions K is represented by a term in the sum Σ in which m_κ is the dielectric constant of the K ions. Then $1 + \Sigma m_\kappa$ is the observed dielectric constant D of the substance for infinitely long waves. If there are two families of resonance ions, one producing the ultra-violet absorption band of wave length λ_v and the other responsible for the infra-red absorption band of wave length λ_r, the dispersion is represented by

$$n^2 = 1 + \frac{m_v \lambda^2}{\lambda^2 - \lambda_v^2} + \frac{m_r \lambda^2}{\lambda^2 - \lambda_r^2}$$

Miss Howell [59] gave a Sellmeier formula for air having an infra-red absorption band of wave length 31 800 A, "probably due to traces of water vapor," but this seems very doubtful.

If there is only one family of resonance ions, the formula reduces to

$$n^2 = 1 + \frac{m_v \lambda^2}{\lambda^2 - \lambda_v^2}$$

[57] Tangle, Ann. d. Phys. (4), 26, p. 59; 1908.
[58] Schmidt, Ann. d. Phys. (4), 11, p. 121; 1903.
[59] Howell, Phys. Rev., 6, p. 81; 1915.

and this first term of Sellmeier's formula gives a satisfactory representation of the dispersion of all transparent substances in which the infra-red absorption is negligible.

The observational equations previously described were solved by the method of least squares and gave the following expression for the dispersion of air at 0° C and 760 mm

$$n^2 = 1 + \frac{0.00057378\lambda^2}{\lambda^2 - 595{,}260}$$

This simple Sellmeier dispersion formula represents the observations nearly as well as the Cauchy formula given above.

VIII. SUMMARY

A survey of previous researches on refraction of air shows that many investigators have worked either with white light or with one monochromatic radiation, and dispersion measurements have been limited to a small interval of the spectrum. No index measurements exist for waves longer than those corresponding to orange light, and in the ultra-violet the dispersion formulae disagree by more than 10 per cent of the refractivity.

Recent work in spectroscopy makes it very desirable to have more accurate and extensive data on the index of refraction and dispersion of air. The international system of standard wave lengths expresses the lengths of waves in air at 15° C and 760 mm, and all wave-length measurements made under other conditions require small corrections because of the effect of temperature and pressure of the air upon its optical dispersion. Furthermore, it is often desirable to multiply wave lengths measured in air by the indices of refraction of air for these wave lengths and thus convert them to their value in a vacuum. An accuracy of one part in several millions is now striven for in the measurement of wave lengths, and to maintain their relative accuracy in the values reduced to vacuum it is necessary to know the indices of refraction within a few units in the seventh decimal place.

For several years this Bureau has been engaged in the accurate measurement of wave lengths. Interferometer comparisons of standard wave lengths have been made throughout a large range of spectrum and the grating spectra of more than 50 of the chemical elements have been photographed and measured in the red and infra-red spectral regions. In connection with these accu-

rate measurements of wave lengths it was thought advisable to measure the absolute indices of refraction of air for the entire spectral region which is accessible to photography.

Accuracy and efficiency recommended the use of the Fabry and Perot interferometer for this work, since this apparatus can be conveniently inclosed in a chamber in which the temperature and pressure of the air can be regulated as desired, and it also permits simultaneous observations for a large number of different wave lengths. Sections of the circular fringes, produced by various radiations from a source of light illuminating the parallel plates of the interferometer, were photographed either with a grating or a rock-salt prism spectrograph, first when the space between the plates was evacuated, and then when dry air at measured temperature and pressure was present.

The index of refraction for a particular wave length was obtained directly from measurements of the photographed interference fringes which allowed the ratio of lengths of this wave in vacuum and in air to be calculated. Observations were made at spectrum intervals of about 40 A from the extreme ultra-violet at 2200 A, through the visible spectrum and in the infra-red to 9000 A.

Complete sets of observations were made on dry air at atmospheric pressure and at temperatures of 0, 15, and 30° C. These are quite closely represented by the following dispersion formulas of the Cauchy form:

$$(n-1)_0 \times 10^7 = 2875.66 + \frac{13.412}{\lambda^2 \times 10^{-8}} + \frac{0.3777}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{15} \times 10^7 = 2726.43 + \frac{12.288}{\lambda^2 \times 10^{-8}} + \frac{0.3555}{\lambda^4 \times 10^{-16}}$$

$$(n-1)_{30} \times 10^7 = 2589.72 + \frac{12.259}{\lambda^2 \times 10^{-8}} + \frac{0.2576}{\lambda^4 \times 10^{-16}}$$

These observations are used in the construction of a table giving the corrections which must be applied to wave lengths measured in air whose density is not normal. A table of corrections to convert wave lengths or frequencies measured in air to their values in a vacuum is also given.

The coefficient of index variation with temperature was found from these measurements to be a function of the wave length. For long waves this optical temperature coefficient is identical with the density temperature coefficient—that is, $\frac{1}{273}$—but as the

ultra-violet absorption band is approached it increased rapidly, becoming $\frac{1}{258}$ at 2500 A.

There seems to be no definite evidence of any strong absorption of infra-red light by dry air and it is, therefore, possible to represent the optical dispersion of air by the first term of Sellmeier's formula quite satisfactorily.

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